Denavit and Hartenberg (DH) Parameters

(Excerpt from Chapter 5 of the book "Introduction to Robotics" by S.K. Saha, Tata McGraw-Hill, New Delhi, 2008)

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5.4 Denvit and Hartenberg (DH) Parameters

A robot manipulator consists of several links connected by, usually, single degree of freedom joints, say, a revolute or a prismatic joint. In order to control the end-effector with respect to the base, it is necessary to find the relation between the coordinate frames attached to the end-effector and the base. This can be obtained from the description of the coordinate transformations between the coordinate frames attached to all the links and forming the overall description in a recursive manner. For this purpose, the material presented in the previous section for describing the position and orientation of the rigid body is useful for obtaining composition of coordinate transformations between the consecutive frames. As a first step, a systematic general method is to be derived to define the relative position and orientation of two consecutive links. The problem is to define two frames attached to two successive links and compute the coordinate transformation between them. In general, the frames are arbitrarily chosen as long as they are attached to the link they are referred to. Nevertheless, it is convenient to set some rules for the definition of the link frames. The convention adopted here for a serial chain robot shown in Fig. 5.22 is that it has n + 1 links, namely, link #0, . . . #n, coupled by n joints, i.e., joint 1, . . . n.

> *First appearance of DH parameters*
>
> The DH parameters were first appeared in 1955 (Denavit and Hartenberg, 1955) to represent a directed line which is nothing but the axis of a lower pair joint.

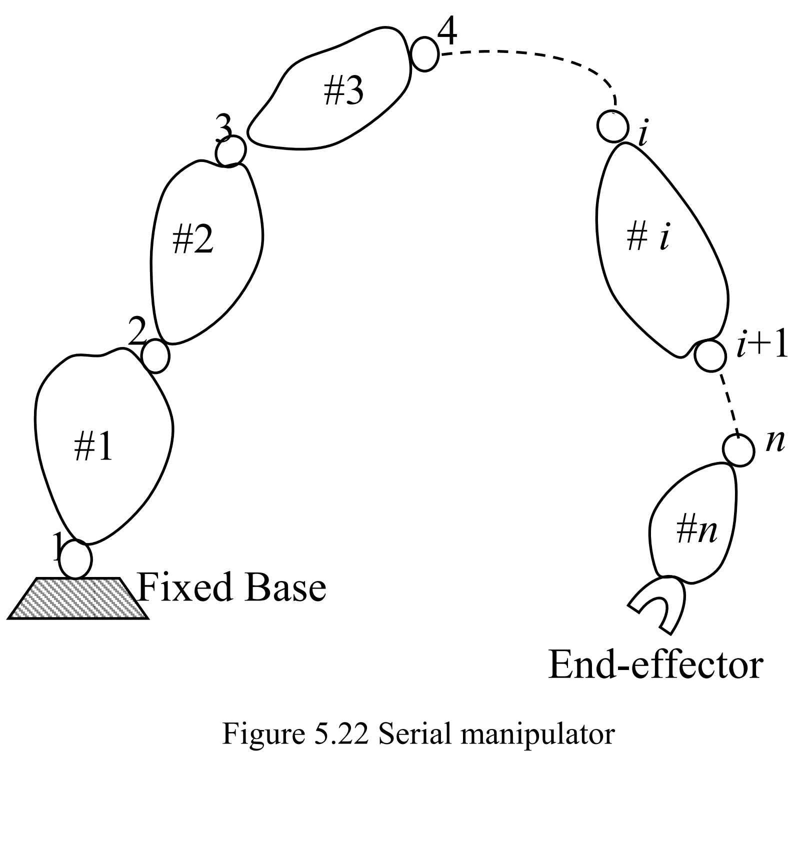


Figure 5.22 Serial manipulator

Now, referring to Fig. 5.23,

(a) Let axis i denotes the axis of the joint connecting link $i-1$ to link i.

(b) A coordinate system X_i, Y_i, Z_i is attached to the end of the link $i-1$ — not to the link i! — for $i = 1, \ldots n+1$.

(c) Choose axis Z_i along the axis of joint i, whose positive direction can be taken towards either direction of the axis.

(d) Locate the origin, O_i, at the intersection of axis Z_i with the common normal to Z_{i-1} and Z_i. Also, locate O'_i on Z_i at the intersection of the common normal to Z_i and Z_{i+1}.

(e) Choose axis X_i along the common normal to axes Z_{i-1} and Z_i with the direction from former to the later.

(f) Choose axis Y_i so as to complete a right handed frame.

Note that the above conventions do not give a unique definition of the link frames in the following cases:

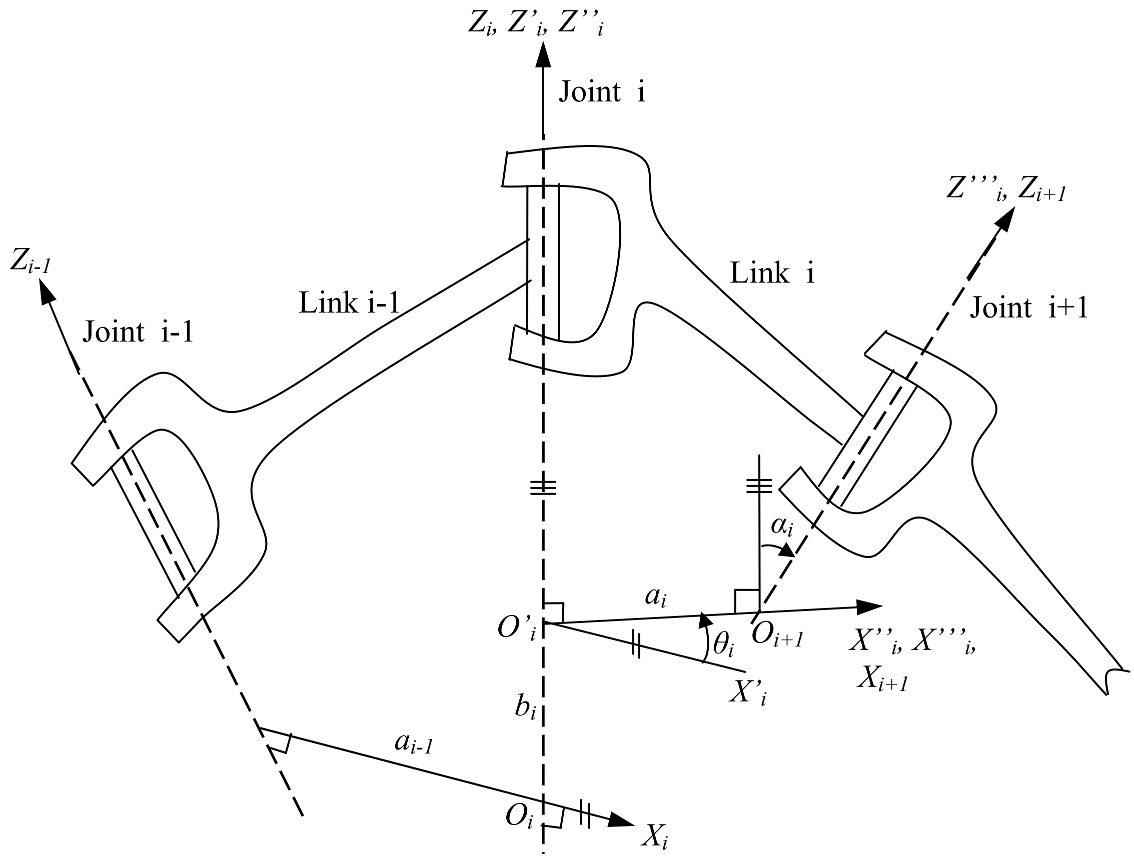
- For frame 1 that is attached to the fixed base, i.e., link 0, only the direction of axes Z_1 is specified. Then O_1 and X_1 can be chosen arbitarily.

- For the last frame $n+1$ the foregoing convention do not apply since there is no link $n+1$. Thus, frame $n+1$ can be arbitrarily chosen.

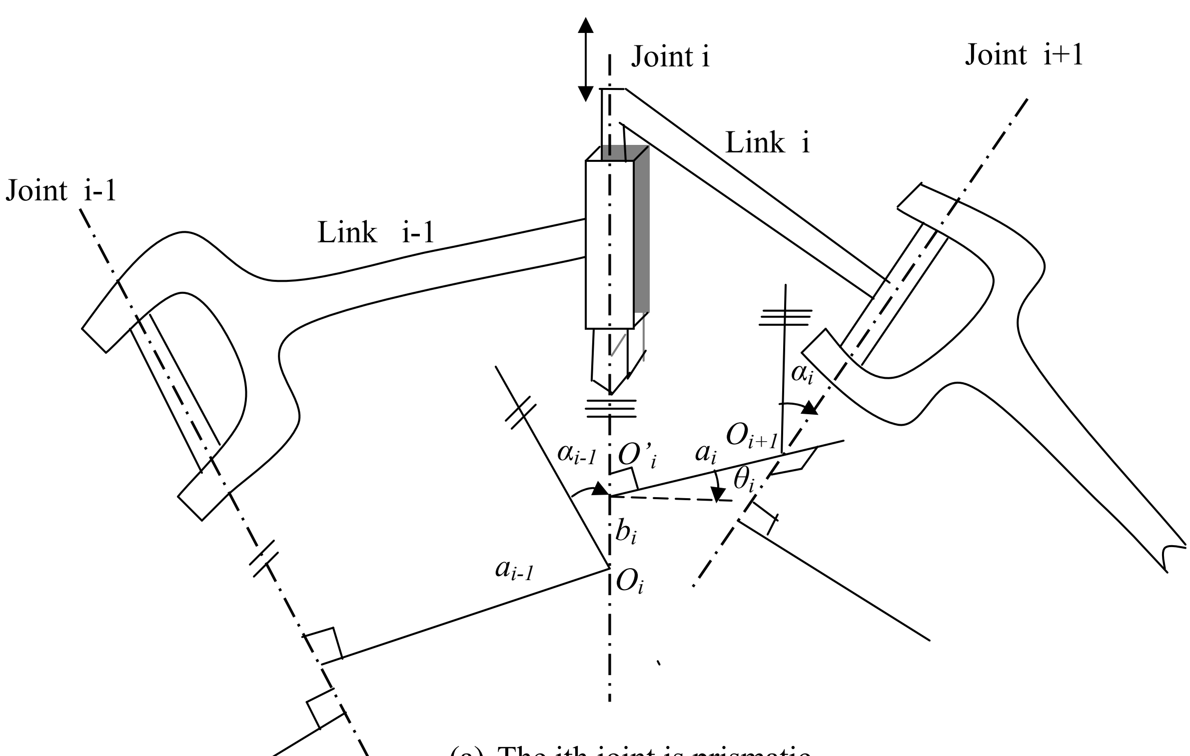
- When two consecutive axes are parallel, the common normal between them is not uniquely defined.

- When two consecutive axes intersect, the direction of X_i is arbitrary.

When joint i is prismatic, only the direction of axis Z_i is determined, whereas the location of O_i is arbitrary.



(a) The ith joint is revolute



(a) The ith joint is prismatic

Figure 5.23 Frame convention and Denavit and Hartenberg (DH) parameters

In all such cases, the indeterminacy can be exploited to simplify the procedure. For instance, the axes of frame $n + 1$ can be made parallel to those of frame n. Once the link frames have been established, the position and orientation of frame i with respect to frame $i - 1$ are completely specified by four parameters known as the Denavit and Hartenberg (DH) parameters. Hence, these frames are also referred as DH frames. The four DH parameters are defined as follows:

(a) b_i (Joint offset)

Length of the intersections of the common normals on the joint axis Z_i, i.e., O_i and O'_i. It is the relative position of links $i - 1$ and i. This is measured as the distance between X_i and X_{i+1} along Z_i.

(b) θ_i (Joint angle)

Angle between the orthogonal projections of the common normals, X_i and X_{i+1}, to a plane normal to the joint axes Z_i. Rotation is positive when it is made counter clockwise. It is the relative angle between links $i - 1$ and i. This is measured as the angle between X_i and X_{i+1} about Z_i.

(c) a_i (Link length)

Length between the O'_i and O_{i+1}. This is measured as the distance between the common normals to axes Z_i and Z_{i+1} along X_{i+1}.

(d) α_i (Twist angle)

Angle between the orthogonal projections of joint axes, Z_i and Z_{i+1} onto a plane normal to the common normal. This is measured as the angle between the axes, Z_i and Z_{i+1}, about axis X_{i+1} to be taken positive when rotation is made counter clockwise.

Note that the above four parameters are defined sequentially as one moves from link $i - 1$ to link $i + 1$ though link i. Moreover, the first two parameters, namely, b_i and θ_i, define the relative position of links $i - 1$ and i, whereas the last two, a_i and α_i, describe the size and shape of link i that are always constant. Parameters, b_i and θ_i, are, however, variable depending on the type of joints in use. In particular,

- θ_i is variable if joint i is revolute; and
- b_i is variable if joint i is prismatic.

So, for a given type of joint, i.e., revolute or prismatic, one of the DH parameters is variable, which is called 'joint variable,' whereas the other three remaining parameters are constant that are called 'link parameters.'

Example 5.16 DH Parameters of a Three-link Planar Arm

Figure 5.24 shows a three link planar arm. The coordinate frames to define the DH parameters are shown in the figure. The DH parameters are tabulated in Table 5.2, where a_i and θ_i, for $i = 1, 2, 3$, are the link lengths and the joints angles, respectively. Axis Z_i is perpendicular to the plane of page and X_1 is chosen arbitrarily. Note that Frame 1, i.e., X_1, Y_1, and Z_1, is fixed to link denoted as #0. Since there is no link 4, frame 4 can be arbitrarily assigned so that its X-axis is placed along the link, as done for frames 2 and 3.

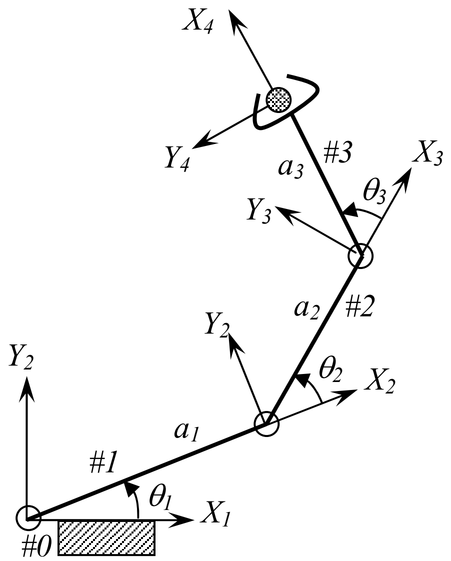


Figure 5.24 A three-link planar arm

Table 5.2 DH parameters of the three-link arm

Link	b_i	θ_i	a_i	α_i
1	0	θ_1 (JV)	a_1	0
2	0	θ_2 (JV)	a_2	0
3	0	θ_3 (JV)	a_3	0

JV: Joint Variable

Note that for a 2-link planar with both revolute joints, i.e., #3 is removed from Fig. 5.24, the DH parameters of Table 5.2 without the third row hold good.

Example 5.17 DH Parameters of a Revolute-Prismatic Planar Arm

Referring to a Revolute-Prismatic (RP) planar arm, Fig. 5.25, where the revolute and prismatic joints are indicated as R and P, respectively, the DH parameters are listed in Table 5.3.

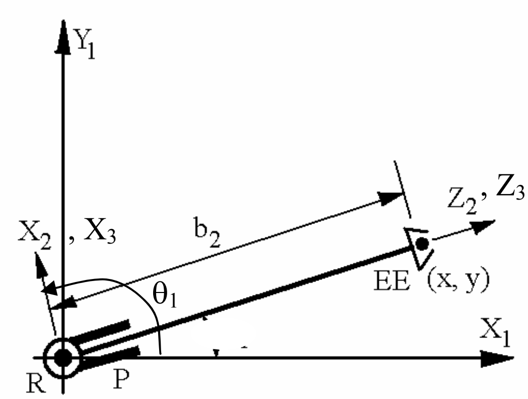


Figure 5.25 Revolute-Prismatic planar arm

Table 5.3 DH parameters of the RP arm

Link	b_i	θ_I	a_i	α_i
1	0	θ_1 (JV)	0	π/2
2	b_2 (JV)	0	0	0°

Example 5.18 DH Parameters of a Prismatic-Revolute Planar Arm

If the revolute and prismatic joints are interchange, the result is a Prismatic-Revolute (PR) arm, as shown in Fig. 5.26. Its DH parameters are shown in Table 5.4.

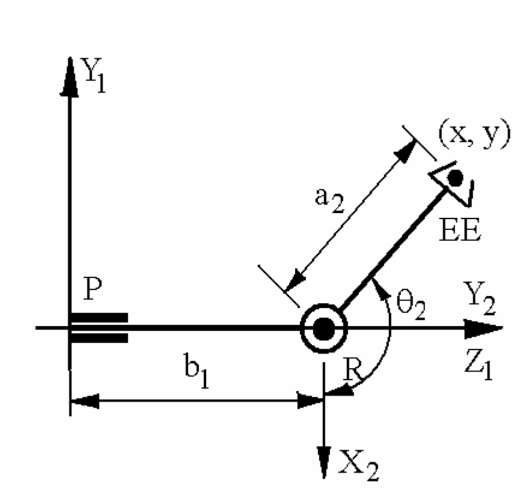


Figure 5.26 Prismatic-Revolute planar arm

Table 5.4 DH parameters of the PR arm

Link	b_i	θ_I	a_i	α_i
1	b_2 (JV)	0	0	π/2
2	0	θ_2 (JV)	a_2	π/2

Example 5.19 DH Parameters of a Spherical Arm

Referring to the spherical type robot arm shown in Fig. 5.27, note that the first and second links, namely, #1 and #2, intersect and the first link length does not affect the end-effector motion due to the rotation of the first joint. So it is beneficial to put both the first and second frames at the intersection of the first two revolute axes, namely, at O_1 or O_2. The DH parameters are tabulated in Table 5.5, where b_2, b_3, and θ_i, for i = 1, 2, 3, are indicated in Fig. 5.27.

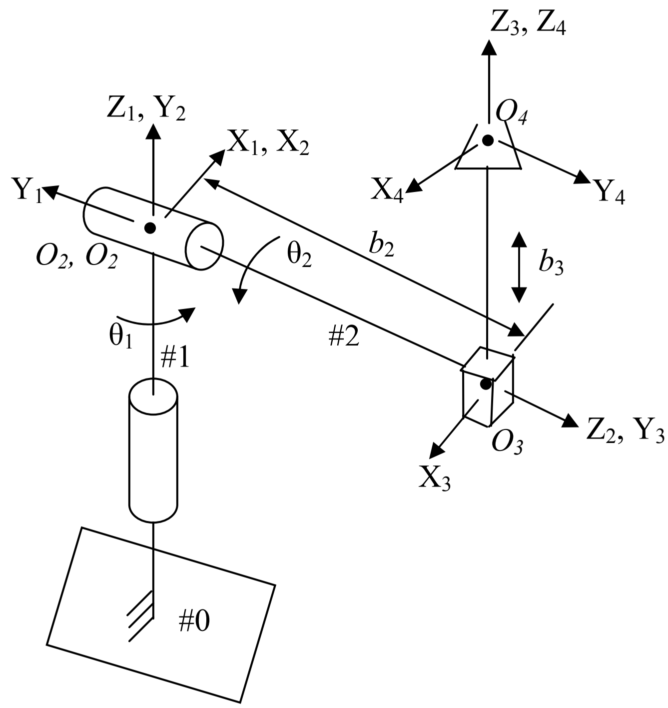


Figure 5.27 A spherical-type arm

Table 5.5 DH parameters of the spherical arm

Link	b_i	θi	a_i	α_i
1	0	θ_1 (JV)	0	$\pi/2$
2	b_2	θ_2 (JV)	0	$\pi/2$
3	b_3 (JV)	0	0	0

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