# RECURSIVE DYNAMICS: APPLICATION 

## TO ROBOTICS, RUR AL MACHINES, ROPES, AND WHHT NEXT?

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## KUKA Robot in RoboAnalyzer




## Free-fall Simulation



0.06

0.06



0.105

0.044837





## Plan of Presentation

- Purpose
- Serial systems
- RoboAnalyzer
- Closed-loop system
- Multibody Dynamics for Rural Applications
- Tree-type system
- Modeling
- Simulation
$\square$ Conclusions


## Modelling and Simulation

## Actually



Mathematically
Newton's $2^{\text {nd }}$ law: $p=m f$
$\rightarrow$ Modelling
Find, $f=p / m ; v=\int f d t, x=\int v d t \rightarrow$ Simulation

## Inverse vs. Forward Dynamics



## Serial Robots

## A Decomposition of the Manipulator Inertia Matrix

Subir Kumar Saha

Abstract-A decomposition of the manipulator inertia matrix is essential, for example, in forward dynamics, where the joint accelerations are solved from the dynamical equations of motion. To do this, unlike a numerical algorithm, an analytical approach is suggested in this paper. The approach is based on the symbolic Gaussian elimination of the inertia matrix that reveal recursive relations among the elements of the resulting matrices. As a result, the decomposition can be done with the complexity of order $n, \mathcal{O}(n)-n$ being the degrees of freedom of the manipulator-, as opposed to an $\mathcal{O}\left(n^{3}\right)$ scheme, required in the numerical approach. In turn, $\mathcal{O}(n)$ inverse and forward dynamics algorithms can be developed. As an illustration, an $\mathcal{O}(n)$ forward dynamics algorithm is presented.

Index Terms- Articulated-body inertia, Kalman filtering, reverse Gaussian elimination (RGE), serial manipulator, symbolic decomposition.

## I. Introduction

The inertia matrix of a robotic manipulator or the generalized inertia matrix (GIM) arises from the robot's dynamic equations of motion. The decomposition of the GIM is required, for example,

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## 1997 IEEE Trans. on Rob. \& Aut.

 V. 13, N. 2, Apr., pp. 301-304
## PUMA Robot

## Methods

- Newton-Euler (NE)

Euler's: $\mathbf{I}_{i} \dot{\omega}_{i}+\omega_{i} \times \mathbf{I}_{i} \omega_{i}=\mathbf{n}_{i}$

Newton's:

$$
m_{i} \dot{\mathbf{v}}_{i}=\mathbf{f}_{i}
$$

$$
\mathbf{M}_{i} \dot{\mathbf{t}}_{i}+\dot{\mathbf{M}}_{i} \mathbf{t}_{i}=\mathbf{w}_{i}
$$

- Euler-Lagrange (EL)

$$
\frac{d}{d t}\left(\frac{\partial L}{\partial \dot{\boldsymbol{\theta}}}\right)-\frac{\partial L}{\partial \boldsymbol{\theta}}=\boldsymbol{\tau}
$$

- Kane's, Hamilton's ...
- Orthogonal Complement based, e.g., Decoupled Natural Orthogonal Complement (DeNOC)



## Uncoupled NE Equations


-The $6 n$ uncoupled equations of motion

## Kinematic Constraints: DeNOC Matrices



$$
\begin{aligned}
& \omega_{i}=\omega_{j}+\dot{\theta}_{i} \mathbf{e}_{i} \\
& \mathbf{v}_{i}=\mathbf{v}_{j}+\omega_{j} \times \mathbf{r}_{j}+\omega_{i} \times \mathbf{d}_{i} \\
& \mathbf{t}_{i}=\mathbf{B}_{i j} \mathbf{t}_{j}+\mathbf{p}_{i} \dot{\theta}_{i} \\
& \\
& \mathbf{B}_{i j} \mathbf{B}_{j k}=\mathbf{B}_{i k} \\
& \mathbf{B}_{i i}=\mathbf{1}, \quad \text { and } \quad \mathbf{B}_{i j}^{-1}=\mathbf{B}_{j i}
\end{aligned}
$$

$\mathbf{B}_{i j}$ : the $6 n \times 6 n$ twist-propagation matrix
$\mathbf{p}_{i}$ : the $6 n$-dimensional joint-rate propagation vector or twist generator

## Definition: DeNOC Matrices

$$
\mathbf{t} \equiv\left[\mathbf{t}_{1}^{T}, \cdots, \mathbf{t}_{n}^{T}\right]^{T} \quad \dot{\boldsymbol{\theta}} \equiv\left[\dot{\dot{\theta}}_{1}, \cdots, \dot{\theta}_{n}\right]^{T}
$$

$\mathbf{N}_{l} \equiv\left[\begin{array}{cccc}\mathbf{1} & \mathbf{0} & \cdots & \mathbf{0} \\ \mathbf{B}_{21} & \mathbf{1} & \cdots & \mathbf{0} \\ \vdots & \vdots & \ddots & \vdots \\ \mathbf{B}_{n 1} & \mathbf{B}_{n 2} & \cdots & \mathbf{1}\end{array}\right] \quad$ and $\quad \mathbf{N}_{d} \equiv\left[\begin{array}{cccc}\mathbf{p}_{1} & \mathbf{0} & \cdots & \mathbf{0} \\ 0 & \mathbf{p}_{2} & \cdots & \mathbf{0} \\ \vdots & \vdots & \ddots & \vdots \\ \mathbf{0} & \mathbf{0} & \cdots & \mathbf{p}_{n}\end{array}\right]$

- $\mathbf{N} \equiv \mathbf{N}_{l} \mathbf{N}_{d}$ : the $6 n \times n$ Decoupled Natural Orthogonal Complement


## Coupled Equations

$\mathbf{N}^{T}(\mathbf{M} \dot{\mathbf{t}}+\dot{\mathbf{M}} \mathbf{t})=\mathbf{N}^{T}\left(\mathbf{w}^{W}+\mathbf{w}^{C}\right) \square \mathbf{t}^{T} \mathbf{w}^{C}=\dot{\boldsymbol{\theta}}^{T} \mathbf{N}^{T} \mathbf{w}^{C}=\mathbf{0}$


$$
\mathbf{N}^{T}(\mathbf{M} \dot{\mathbf{t}}+\dot{\mathbf{M}} \mathbf{t})=\mathbf{N}^{T} \mathbf{w}^{W}
$$



$$
\mathrm{I} \ddot{\theta}+\mathrm{C} \dot{\theta}=\tau
$$

$$
\begin{aligned}
\mathbf{I} & \equiv \mathbf{N}^{T} \mathbf{M} \mathbf{N} \equiv \mathbf{N}_{d}^{T} \tilde{\mathbf{M}} \mathbf{N}_{d} \\
\mathbf{C} & \equiv \mathbf{N}^{T}\left(\mathbf{M} \mathbf{N}+\stackrel{\ddot{\mathbf{M}} \mathbf{N}) \equiv \mathbf{N}_{d}^{T} \tilde{\mathbf{M}}^{\prime} \mathbf{N}_{d}}{\boldsymbol{\tau}} \equiv \begin{array}{|}
\mathbf{N}^{T} \mathbf{w}^{W} \equiv \mathbf{N}_{d}^{T} \tilde{\mathbf{w}}^{W}
\end{array}\right.
\end{aligned}
$$

- $n$ coupled Euler-Lagrange equations
- no partial differentiation


## Recursive Expressions

- For the $n \times n$ GIM, each element

$$
I_{i j}=I_{j i}=\mathbf{p}_{i}^{T} \tilde{\mathbf{M}}_{i} \mathbf{B}_{i j} \mathbf{p}_{j}
$$

$$
\begin{aligned}
& \overline{\mathbf{M}}_{i}=\mathbf{M}_{i}+\mathbf{B}_{i+1, i}^{T} \tilde{\mathbf{M}}_{i+1} \mathbf{B}_{i+1, i} \text { where } \tilde{\mathbf{M}}_{n} \equiv \mathbf{M}_{n} \\
& \text { Composite body mass matrix }
\end{aligned}
$$

- For the $n \times n \mathrm{MCI}$, each element

$$
C_{i j}= \begin{cases}\mathbf{p}_{i}^{T}\left(\mathbf{B}_{j i}^{T} \tilde{\mathbf{M}}_{j} \mathbf{W}_{j}+\mathbf{B}_{j+1, i}^{T} \tilde{\mathbf{H}}_{j+1, j}+\overline{\mathbf{M}}_{j}\right) \mathbf{p}_{j} & \text { if } i \leq \tilde{\mathbf{M}}_{n+1}=\tilde{\mathbf{H}}_{n+1, n}=\mathbf{O} \\ \mathbf{p}_{i}^{T}\left(\tilde{\mathbf{M}}_{i} \mathbf{B}_{i j} \mathbf{W}_{j}+\tilde{\mathbf{H}}_{i j}+\dot{\mathbf{M}}_{i}\right) \mathbf{p}_{j} & \text { otherwise }\end{cases}
$$

- For the $n \times n$ generalized forces

$$
\tau_{i}=\mathbf{p}_{i}^{T} \tilde{\mathbf{w}}_{i}^{W} \quad \tilde{\mathbf{w}}_{i}^{W}=\mathbf{w}_{i}^{W}+\mathbf{B}_{i+1, i}^{T} \tilde{\mathbf{w}}_{i+1}^{W}
$$

## Inverse Dynamics Algorithm

$$
\begin{aligned}
& \text { Forward Recursion } \\
& \nu_{1}=\mathbf{p}_{1} \dot{\theta}_{1} \text {; } \\
& \nu_{2}=\mathbf{p}_{2} \dot{\theta}_{2}+\nu_{1} ; \\
& \text {; } \\
& \nu_{n}=\mathbf{p}_{n} \dot{\theta}_{n}+\nu_{n-1} ; \\
& \xi_{1}=\mathbf{p}_{1} \ddot{\theta}_{1}+\mathbf{W}_{1} \mathbf{p}_{1} \dot{\theta}_{1} \\
& \boldsymbol{\xi}_{2}=\mathbf{p}_{2} \ddot{\theta}_{2}+\mathbf{W}_{2} \mathbf{p}_{2} \dot{\theta}_{2}+\mathbf{B}_{21} \xi_{1}+\dot{\mathbf{B}}_{21} \nu_{1} \\
& \boldsymbol{\xi}_{n}=\mathbf{p}_{n} \ddot{\theta}_{n}+\mathbf{W}_{n} \mathbf{p}_{n} \dot{\theta}_{n}+\mathbf{B}_{n, n-1} \xi_{n-1} \\
& +\dot{\mathbf{B}}_{n, n-1} \boldsymbol{\nu}_{n-1} \\
& \tau_{n}=\mathbf{p}_{n}^{T} \gamma_{n} \\
& \tau_{n-1}=\mathbf{p}_{n-1}^{T} \gamma_{n-1} \\
& ; \\
& \tau_{1}=\mathbf{p}_{1}^{T} \gamma_{1}
\end{aligned}
$$

## Example: PUMA 560



## Result: Torque at Joint 1

$\theta_{i}=\frac{1}{2}\left[\frac{2 \pi}{T} t-\sin \left(\frac{2 \pi}{T} t\right)\right] T=10.0 \mathrm{sec} ; \theta_{i}(0)=0$, and $\theta_{i}(T)=180^{\circ}$


## Comparison

| Algorithm | $M$ | $A$ | $n=6$ |  |
| :--- | :---: | :---: | :---: | ---: |
| Hollerbach (1980) | $412 n-277$ | $320 n-201$ | $2195 M$ | $1719 A$ |
| Luh et al. (1980) | $150 n-48$ | $131 n+48$ | $852 M$ | $834 A$ |
| Walker and Orin (1982) | $137 n-22$ | $101 n-11$ | $800 M$ | $595 A$ |
| Proposed | $\frac{120 n-44}{105 n-92}$ | $\underline{97 n-55}$ | $94 n-86$ | $\frac{676 M}{538 M}$ |$\underline{\underline{527 A}} 478 A$

## Forward Dynamics \& Simulation



$$
\begin{array}{cc}
\hat{\tau}_{i}=\phi_{i}-\mathbf{p}_{i}^{T} \eta_{i, i+1} & \eta_{i, i+1} \equiv \mathbf{B}_{i+1, i}^{T} \boldsymbol{\eta}_{i+1} \quad \eta_{i+1} \equiv \psi_{i+1} \hat{\tau}_{i+1}+\eta_{i+1, i+2} \quad \hat{\tau}_{n} \equiv \phi_{n} \\
\ddot{\theta}_{i}=\tilde{\tau}_{i}-\psi_{i}^{T} \mu_{i, i-1} & \mu_{i-1} \equiv \mathbf{p}_{i-1} \ddot{\theta}_{i-1}+\mu_{i-1, i-2} \quad \mu_{i, i-1} \equiv \mathbf{B}_{i, i-1} \mu_{i-1} \quad \mu_{1,0}=\mathbf{0}
\end{array}
$$

$\hat{\boldsymbol{\psi}}_{k} \equiv \hat{\mathbf{M}}_{k} \mathbf{p}_{k}$,

$$
\hat{\psi}_{i k} \equiv \mathbf{B}_{k i}^{T} \hat{\psi}_{k}
$$

$$
1 \equiv \mathbf{p}_{k}^{2} \boldsymbol{\psi}_{k} \text {, }
$$

$$
\psi_{k} \equiv \frac{\hat{\psi}_{k}}{\hat{m}_{k}}, \quad \text { and } \quad \psi_{i k} \equiv \frac{\hat{\psi}_{i k}}{\hat{m}_{k}}
$$

$$
\hat{\mathbf{M}}_{i k}=\mathbf{M}_{i}+\mathbf{B}_{i+1, i}^{T} \hat{\mathbf{M}}_{i+1, k} \mathbf{B}_{i+1, i}
$$

## Comparison

| Algorithm | $M$ | $A$ |
| :---: | :---: | :---: |
| Proposed | $\frac{191 n-284}{199 n-198}$ | $\frac{187 n-325}{174 n-173}$ |
| Featherstone (1983) | $226 n-343$ | $206 n-345$ |
| Valasek $\dagger$ | $250 n-222$ | $220 n-198$ |
| Brandl et al. $\dagger$ | $\frac{1}{6} n^{3}+11 \frac{1}{2} n^{2}$ | $\frac{1}{6} n^{3}+7 n^{2}$ |
| Walker and Orin (1982) | $+38 \frac{1}{3} n-47$ | $+38 \frac{5}{6} n-46$ |


| $n=6$ | $n=10$ |
| :---: | :---: |
| $\underline{862 M 797 A}$ | $\underline{1626 M 1545 A}$ |
| $996 M 871 A$ | $1792 M 1567 A$ |
| $1013 M 891 A$ | $1917 M 1715 A$ |
| $1278 M 1122 A$ | $2278 M 2002 A$ |
| $633 M 480 A$ | $1653 M 1209 A$ |

## Results:

## Stanford Robot

## DH and Inertia Parameters



| $i$ | $a_{i}$ | $b_{i}$ | $\alpha_{i}$ | $\theta_{i}$ | $m_{i}$ | $r_{x}$ | $r_{y}$ | $r_{z}$ | $I_{x x}$ | $I_{y y}$ | $I_{z z}$ |
| :---: | :---: | :---: | :---: | :---: | :---: | :---: | :---: | :---: | :---: | :---: | :---: |
|  | $(\mathrm{~m})$ | $(\mathrm{m})$ | $(\mathrm{deg})$ | $(\mathrm{deg})$ | $(\mathrm{kg})$ | $(\mathrm{m})$ |  |  | $\left(\mathrm{kg}-\mathrm{m}^{2}\right)$ |  |  |
| 1 | 0 | .1 | -90 | $\theta_{1}[0]$ | 9 | 0 | -.1 | 0 | .01 | .02 | .01 |
| 2 | 0 | .1 | -90 | $\theta_{2}[90]$ | 6 | 0 | 0 | 0 | .05 | .06 | .01 |
| 3 | 0 | $b_{3}[0]$ | 0 | 0 | 4 | 0 | 0 | 0 | .4 | .4 | .01 |
| 4 | 0 | .6 | 90 | $\theta_{4}[0]$ | 1 | 0 | -.1 | 0 | .001 | .001 | .0005 |
| 5 | 0 | 0 | -90 | $\theta_{5}[0]$ | .6 | 0 | 0 | 0 | .0005 | .0005 | .0002 |
| 6 | 0 | 0 | 0 | $\theta_{6}[0]$ | .5 | 0 | 0 | 0 | .003 | .001 | .002 |

## Free-Fall: Joints 2 and 3



Robot Software: RoboAnalyzer www.roboanalyzer.com

## Why UDU ${ }^{\text {T? }}$

- Accurate (based on Reverse Gaussian Elimination) [Plot for 3-link system]



## Efficient



## Information on Stability



- Diagonal elements of the GIM


Science, Engineering \& M

## The McGraw Hill Companies




## 

0110010010001 01000110001118 001000010000
1100 INTRODUCCIÓN 001000010001 A0LA 1001101 ROBOTICA 1100100100011
010011000111 Spanish (Mexico)

## 00001000011 SUBIR KUMAR SAHA

 Introduction to

## - dixA:Matheriutioal Fundamentals <br> Chinese <br> (PRC)

vintam anogen nit ose 4ppotiverimiaion


## Carpet Cleaning: Traditional



## Carpet Scrapping Machine

## $\square$ Purpose: To reduce human effort <br> $\square$ Straight line generating machine



## Tree-types: Double Recursion

 ASME J. of Mech. Des., Dec. 2007

- Unknowns: 6+3 \& Eqs. 2+1
- Unsolvable independently (Indeterminate subsystems)
- Unknowns: 4 \& Eqs.: 4
- Solvable independently (determinate subsystem)




## Comparison

## Methods

## Theoretical order of computations

CPU time in sec

Traditional
$\mathrm{O}\left(21^{3 / 3}\right)=\mathrm{O}(3087)$
0.219
(matrix size: $21 \times 21$ )

System approach

$$
0.156
$$ (matrix size: $7 \times 7$ )

$$
\begin{equation*}
\mathrm{O}\left(7^{3} / 3\right)+\mathrm{O}(7 \times 2)=\mathrm{O}(128.3) \tag{30.59}
\end{equation*}
$$

Subsystem approach (matrix sizes:
$\mathrm{O}\left(4^{3} / 3+3^{3} / 3+1\right)+\mathrm{O}(7 \times 2)=\mathrm{O}(45.3)$
0.156
$4 \times 4,3 \times 3,1 \times 1$ )

## Multibody Dynamics for Rural Applications (MuDRA)

- Areas of Research: Multibody Dynamics
- Robotics
- Mechatronics
- Design
- Developed Mechanisms: Rural Applications
- For carpet processing
- For villages ADPM


## MuDRA Concept

- Floated as B. Tech/M. Tech Projects $■$ Not interested
- Apparent Reasons

■ What is the research content ?

- Not fashionable
- Other Reasons
- Limited literature
- Difficult


## springer.com



## Dynamics and Balancing of Multibody Systems

Series: Lecture Notes in Applied and Computational Mechanics . Vol. 37
Chaudhary. Himanshu, Saha, Subir Kumar
2009, XIV, 176 p. 59 illus., Hardcover
ISBN: 978-3-540-78178-3
Online version available

Online orders shipping within 2-3 days.
$99,95 €$

This book has evolved from the passionate desire of the anthors in using the modern concepts of multibody dynamics for the design improvement of the machineries used in the rural sectors of India and The World. In this connection, the first author took up his doctoral research in 2003 whose findings have resulted in this book. It is expected that such developments

## Every drop counts

## PEIER FRYMMAN, A STUDENT OF STANFORD UNNEESTIY, HAS INVENIED A MANUFACTURING TECHNOLOGYTO HEP MAKE AFFORDABLE DRIP IRRIGATION FOR SMALL-PLOT FARMERS IN DEVEOPING COUNTRIES. MAUNI SEN REPORTS

Athird of the world's population suffers from water scarcity. Without access to affordable water efficient irrigation, smallplot farmers are unable to grow crops during much of the year.

As part of a Stanford University course in Entrepreneurial Design for Extreme Affordability, Peter Frykman travelled to Ethiopia, where he witnessed first-hand the hardships caused by the worst drought Ethiopla had experienced in 20 years. The farmers he met had
scarce water resources. Locally available drip irrigation products were too expensive and seldom worked properly,

Recognising the need for less costly and more effective ways for smallplot farmers to use their meagre water supplies efficiently, Frykman returned to Stanford and invented a manufacturing technology, and launched his company, Driptech, which makes affordable drip irrigation for small-plot farmers in developing countries.

Describing the technology behind Driptech. Frykman says there are two important parts to the technology. "The first part is the attention we paid to small-scale farmers and how they work that allowed us to design the system specifically to meet their needs. We learned the importance of making our systems simple in order to reduce installation and mainte-


Stantord graduate Peter Frykman explains his invention to farmers in India
nance costs. We learned that highly uniform water application is essential, even with low pressure, so we
down," he elaborates:
The second part is that Driptech is able to manufacture this product anywhere in the world designed Driptech tubing with the very precise,
 using existing plastic bag machinery. "We developed clean, consistent holes that make this possible. We learned that small plots come in many configurations, and made our system modular by designing parts that continue to function well even if it scales up or
small, inexpensive machines that reliably produce tubing to the high level of quality that we require. This 'distributed manufacturing' model allows us to customise products
to meet local needs, minimise transportation costs, and create jobs in nearby communities," adds Frykman.
Compared to flood irrigation, Driptech irrigation increases crop yields by $20 \%$ to $90 \%$, improves product quality, saves water by $30 \%$ to $70 \%$, and reduces the required labour, energy costs for pumping, and fertiliser. Driptech systems operate on very low water pressures as well.
As a graduate student in mechanical engineering at Stanford, Frykman attended the Summer Institute for Entrepreneurship, and he found the programme "exhilarating" and "highly beneficial."

I applied for the programme because I needed help developing my underlying passion for entrepreneurship. I relied on lessons that I had learned during the programme to launch my company Driptech. Even without a formal business background 1 had the confidence to launch my own venture

Currently Frykman and his team are working with farmers in Maharashtra and Karnataka and last year, Driptech launched commercially in India by partnering with one of the largest manufacturing and retail conglomerates in the country. Driptech's systems sell directly to farmers through retail and local distribution channels.

## Ropes (Courtesy: Dr. Suril V. Shah)



## Multibody Systems: ReDySim



WeLD @ IIT Delhi

## Biological Systems: Study of Proteins

- Proteins
(Nanoparticles: 1100 nm ) are part of every cell, tissue, and organ in our bodies.
- Body proteins are constantly being broken down and replaced.
- The protein in the foods we eat is digested into amino acids that are later used to replace these proteins in our bodies.


## Protein

- Proteins are made up of amino acids.
$\square$ Think of amino acids as the building blocks.
- There are 20 different amino acids that join together to make all types of protein.
- Some of these amino acids can't be made by our bodies, so these are known as essential amin 0 acids.
- It's essential that our diet provide these.
"Analysis and Design of Protein Based Nanodevices:
Challenges and Opportunities in Mechanical Design" by
Gregory S. Chirikjian
[ASME J. Mech. Design, July 2005, Vol. 127]
- Each of these amino acid building blocks (monomers) is referred to as a residue.
$\square$ Tens to hundreds of these residues amino acid monomers connect together in a serial manner to create a long chain, known as a polypeptide chain.


## Protein vs. Serial Robot

- From a kinematics point of view, these polypeptide molecules can be considered to be a chain of miniature rigid bodies connected by revolute hinge joints.
$\square$ A protein in its denatured state is a serial linkage with $N+1$ solid links connected by $N$ revolute joint values for $N$ could be as great as several hundred.


## Protein Folding

$\square$ Protein folding is the process by which a protein structure assumes its functional shape or conformation.
$\square$ The folding occurs under the effect of nuclear forces among protein atoms as well as between protein atoms and the solvent's atoms.
$\square$ Can be studied through dynamic simulation

## Result

- Being able to accurately predict the threedimensional structure of a protein based on the known sequences of amino acids in its chain is key to fully understanding a protein's biological functions and thus to manipulating or controlling these functions as a part of disease treatments.


## Activities with Chair Professor Fund

- Presenting new research at ACMD 2010 in Japan
- Research collaboration with IIT Madras on Rope Modeling

■ Supporting several Students/Faculty to visit IIT Delhi and vice-verse

- Will be attending ICMD 2012 in Germany


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$\square$ Mr. Amit Jain
$\square$ Ms. Jyoti Bahuguna
$\square$ Mr. D. Jaitly, and others


## Conclusions

- DeNOC for serial-chain systems
- RoboAnalyzer
- Closed-loop systems and optimization
- MuDRA
- Rope
$\square$ New application to biological systems


## THANK YOU for Your Attention

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