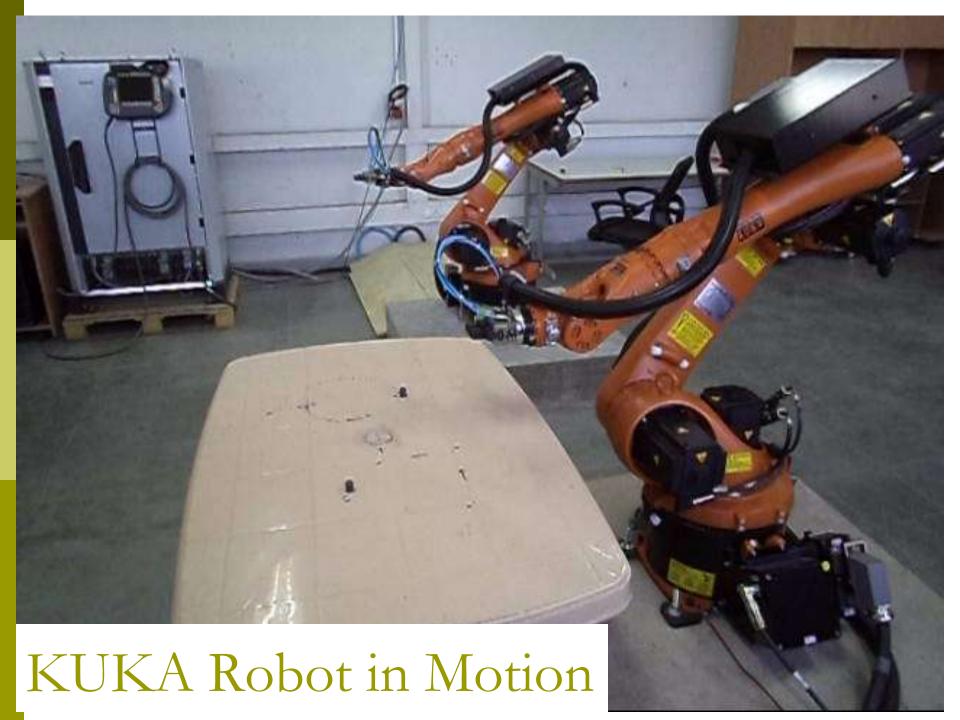
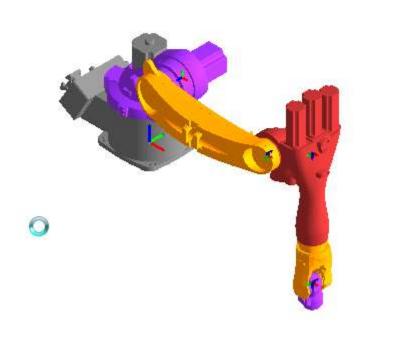
### RECURSIVE DYNAMICS: APPLICATION TO ROBOTICS, RURAL MACHINES, ROPES, AND WHAT NEXT?

#### Prof. S.K. Saha Naren Gupta Chair Professor Dept. of Mech. Eng., IIT Delhi

March 15, 2012

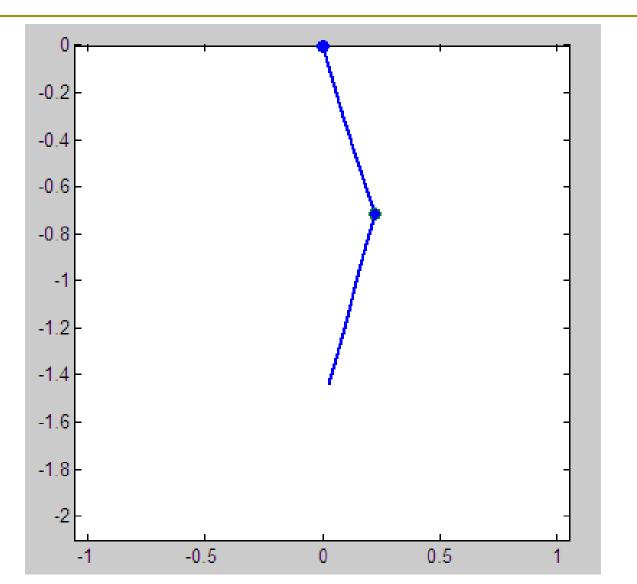


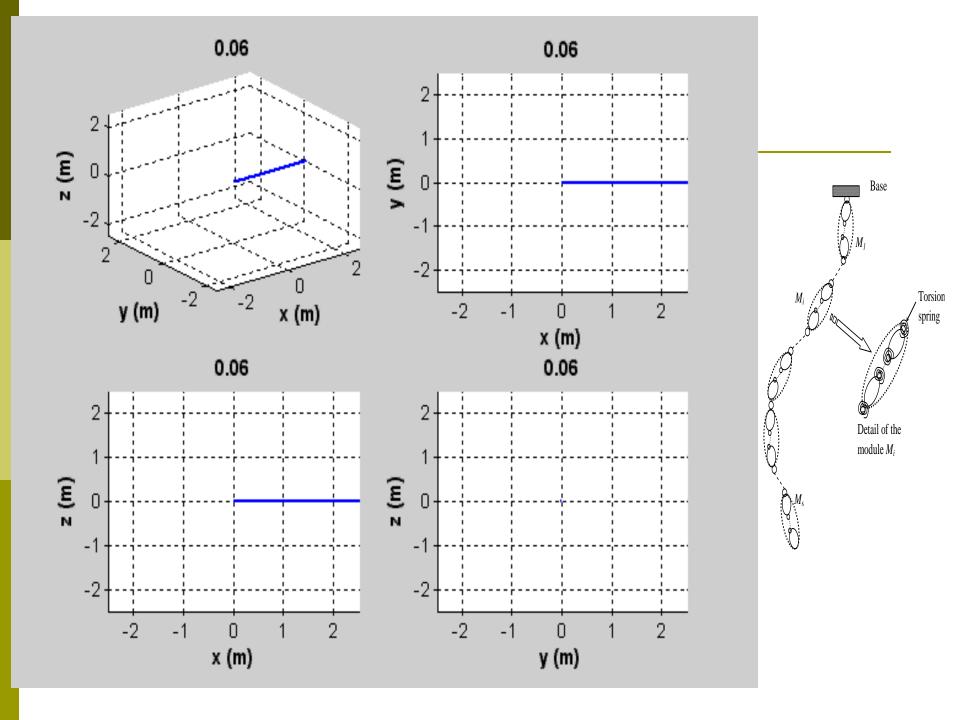
#### KUKA Robot in RoboAnalyzer

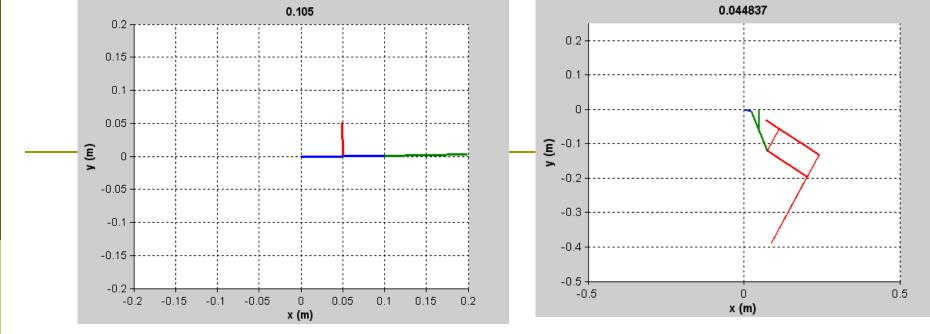


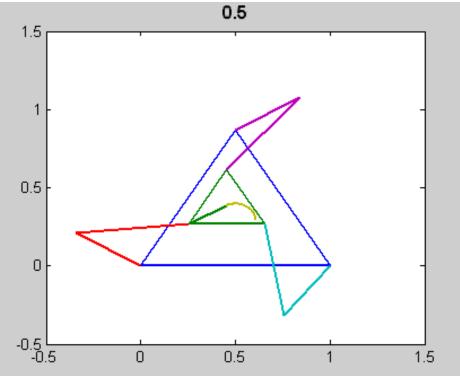


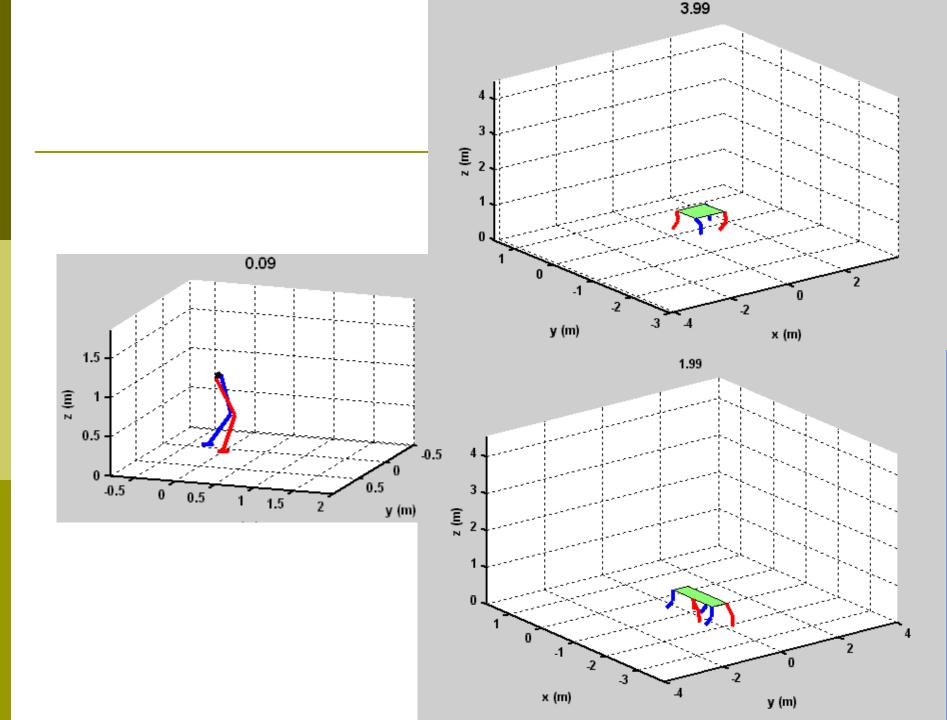
#### Free-fall Simulation











#### Plan of Presentation

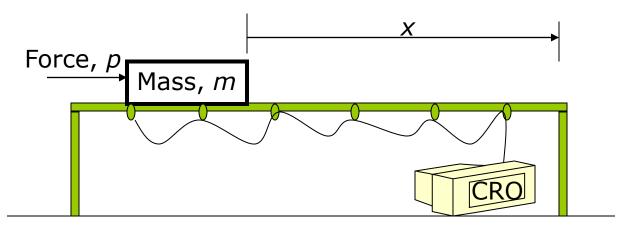
Purpose

- Serial systems
  - RoboAnalyzer
- Closed-loop system
  - Multibody Dynamics for Rural Applications
- Tree-type system
  - Modeling
  - Simulation

Conclusions

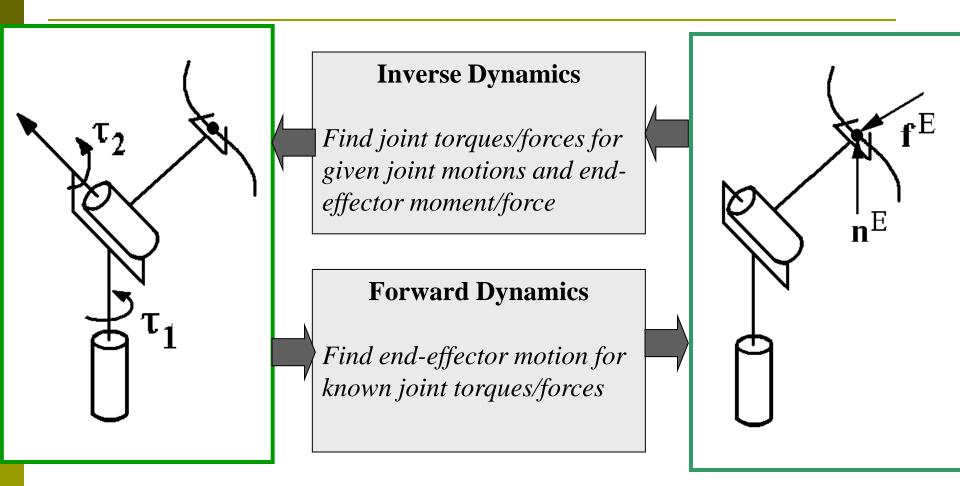
#### Modelling and Simulation

#### Actually



Mathematically Newton's 2<sup>nd</sup> law:  $p = mf \rightarrow$  Modelling Find, f = p/m;  $v = \int f dt$ ;  $x = \int v dt \rightarrow$  Simulation

#### Inverse vs. Forward Dynamics



#### Serial Robots

#### A Decomposition of the Manipulator Inertia Matrix

#### Subir Kumar Saha

Abstract—A decomposition of the manipulator inertia matrix is essential, for example, in forward dynamics, where the joint accelerations are solved from the dynamical equations of motion. To do this, unlike a numerical algorithm, an analytical approach is suggested in this paper. The approach is based on the symbolic Gaussian elimination of the inertia matrix that reveal recursive relations among the elements of the resulting matrices. As a result, the decomposition can be done with the complexity of order n,  $\mathcal{O}(n)$ —n being the degrees of freedom of the manipulator—, as opposed to an  $\mathcal{O}(n^3)$  scheme, required in the numerical approach. In turn,  $\mathcal{O}(n)$  inverse and forward dynamics algorithms can be developed. As an illustration, an  $\mathcal{O}(n)$  forward dynamics algorithm is presented.

Index Terms— Articulated-body inertia, Kalman filtering, reverse Gaussian elimination (RGE), serial manipulator, symbolic decomposition.

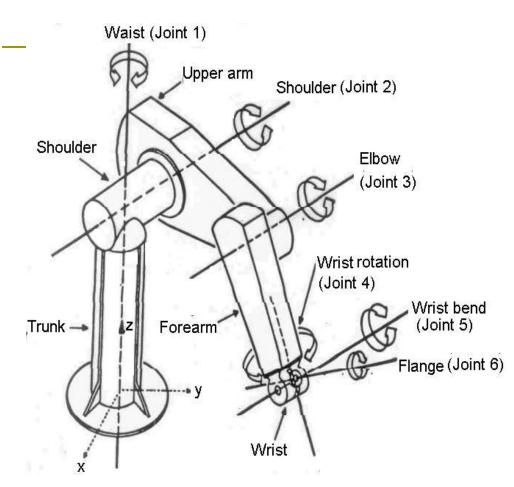
#### I. INTRODUCTION

The inertia matrix of a robotic manipulator or the generalized inertia matrix (GIM) arises from the robot's dynamic equations of motion. The decomposition of the GIM is required, for example,

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**1997 IEEE Trans. on Rob. & Aut.** V. 13, N. 2, Apr., pp. 301-304

## **PUMA Robot**

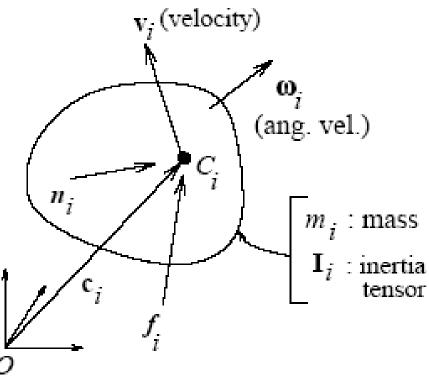
#### Methods

#### Newton-Euler (NE)

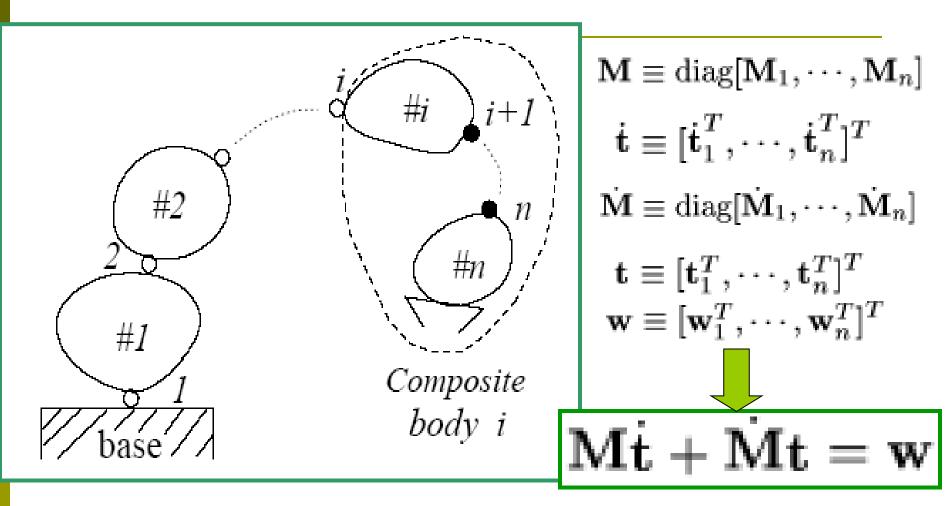
Euler's:
$$\mathbf{I}_i \dot{\omega}_i + \omega_i \times \mathbf{I}_i \omega_i = \mathbf{n}_i$$
Newton's: $m_i \dot{\mathbf{v}}_i = \mathbf{f}_i$  $\mathbf{M}_i \dot{\mathbf{t}}_i + \dot{\mathbf{M}}_i \mathbf{t}_i = \mathbf{w}_i$  $\mathbf{U}_i \mathbf{U}_i \mathbf{U}_i$ 

$$\frac{d}{dt}\left(\frac{\partial L}{\partial \dot{\mathbf{\theta}}}\right) - \frac{\partial L}{\partial \mathbf{\theta}} = \mathbf{\tau}$$

- Kane's, Hamilton's ...
- Orthogonal Complement based, e.g., Decoupled Natural Orthogonal Complement (DeNOC)

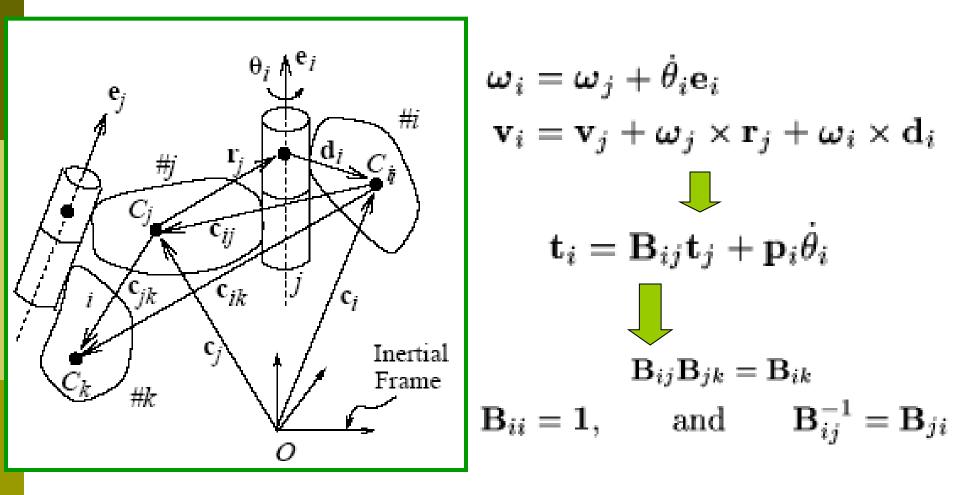


### Uncoupled NE Equations



•The 6*n* uncoupled equations of motion

#### Kinematic Constraints: DeNOC Matrices



 $\mathbf{B}_{ii}$ : the  $6n \times 6n$  twist-propagation matrix

 $\mathbf{p}_i$ : the 6*n*-dimensional joint-rate propagation vector or <u>twist generator</u>

#### Definition: DeNOC Matrices

$$\mathbf{t} \equiv [\mathbf{t}_{1}^{T}, \cdots, \mathbf{t}_{n}^{T}]^{T} \quad \dot{\boldsymbol{\theta}} \equiv [\dot{\theta}_{1}, \cdots, \dot{\theta}_{n}]^{T}$$
$$\mathbf{t} = \mathbf{N}\dot{\boldsymbol{\theta}}, \quad \text{where} \quad \mathbf{N} \equiv \mathbf{N}_{l}\mathbf{N}_{d}$$
$$\mathbf{N}_{l} \equiv \begin{bmatrix} \mathbf{1} & \mathbf{0} & \cdots & \mathbf{0} \\ \mathbf{B}_{21} & \mathbf{1} & \cdots & \mathbf{0} \\ \vdots & \vdots & \ddots & \vdots \\ \mathbf{B}_{n1} & \mathbf{B}_{n2} & \cdots & \mathbf{1} \end{bmatrix} \quad \text{and} \quad \mathbf{N}_{d} \equiv \begin{bmatrix} \mathbf{p}_{1} & \mathbf{0} & \cdots & \mathbf{0} \\ \mathbf{0} & \mathbf{p}_{2} & \cdots & \mathbf{0} \\ \vdots & \vdots & \ddots & \vdots \\ \mathbf{0} & \mathbf{0} & \cdots & \mathbf{p}_{n} \end{bmatrix}$$

•  $N \equiv N_l N_d$ : the  $6n \times n$  Decoupled Natural Orthogonal Complement

#### Coupled Equations

$$\mathbf{N}^{T}(\mathbf{M}\dot{\mathbf{t}} + \dot{\mathbf{M}}\mathbf{t}) = \mathbf{N}^{T}(\mathbf{w}^{W} + \mathbf{w}^{C}) \qquad \mathbf{t}^{T}\mathbf{w}^{C} = \dot{\boldsymbol{\theta}}^{T}\mathbf{N}^{T}\mathbf{w}^{C} = \mathbf{0},$$
$$\mathbf{N}^{T}(\mathbf{M}\dot{\mathbf{t}} + \dot{\mathbf{M}}\mathbf{t}) = \mathbf{N}^{T}\mathbf{w}^{W}$$
$$\mathbf{V}$$
$$\mathbf{V}$$
$$\mathbf{I} \equiv \mathbf{N}^{T}\mathbf{M}\mathbf{N} \equiv \mathbf{N}_{d}^{T}\tilde{\mathbf{M}}\mathbf{N}_{d}$$
$$\mathbf{C} \equiv \mathbf{N}^{T}(\mathbf{M}\dot{\mathbf{N}} + \ddot{\mathbf{M}}\mathbf{N}) \equiv \mathbf{N}_{d}^{T}\tilde{\mathbf{M}}'\mathbf{N}_{d}$$
$$\tau \equiv \mathbf{N}^{T}\mathbf{w}^{W} \equiv \mathbf{N}_{d}^{T}\tilde{\mathbf{w}}^{W};$$

• *n* coupled Euler-Lagrange equations

- no partial differentiation

#### Recursive Expressions

• For the  $n \times n$  GIM, each element  $I_{ij} = I_{ji} = \mathbf{p}_i^T \tilde{\mathbf{M}}_i \mathbf{B}_{ij} \mathbf{p}_j$ 

 $\tilde{\mathbf{M}}_{i} = \mathbf{M}_{i} + \mathbf{B}_{i+1,i}^{T} \tilde{\mathbf{M}}_{i+1} \mathbf{B}_{i+1,i} \text{ where } \tilde{\mathbf{M}}_{n} \equiv \mathbf{M}_{n}$  Composite body mass matrix

• For the  $n \times n$  MCI, each element

$$C_{ij} = \begin{cases} \mathbf{p}_i^T (\mathbf{B}_{ji}^T \tilde{\mathbf{M}}_j \mathbf{W}_j + \mathbf{B}_{j+1,i}^T \tilde{\mathbf{H}}_{j+1,j} + \tilde{\mathbf{M}}_j) \mathbf{p}_j & \text{if } i \leq \tilde{\mathbf{M}}_{n+1} = \tilde{\mathbf{H}}_{n+1,n} = \mathbf{O} \\ \mathbf{p}_i^T (\tilde{\mathbf{M}}_i \mathbf{B}_{ij} \mathbf{W}_j + \tilde{\mathbf{H}}_{ij} + \tilde{\mathbf{M}}_i) \mathbf{p}_j & \text{otherwise} \end{cases}$$

• For the  $n \times n$  generalized forces

$$\tau_i = \mathbf{p}_i^T \tilde{\mathbf{w}}_i^W \qquad \tilde{\mathbf{w}}_i^W = \mathbf{w}_i^W + \mathbf{B}_{i+1,i}^T \tilde{\mathbf{w}}_{i+1}^W$$

# Inverse Dynamics Algorithm

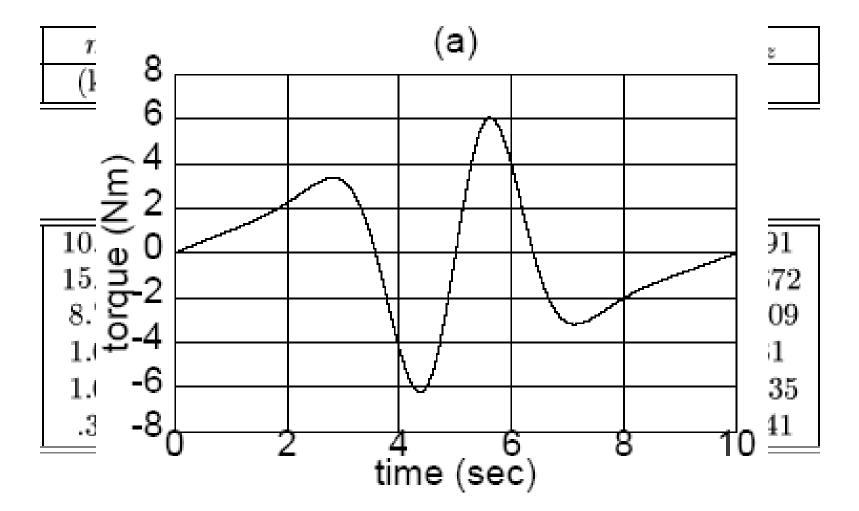
Saha (1999): ASME

### Example: PUMA 560

		DH	Param	eters	
Waist (Joint 1) Upper arm Shoulder (Joint 2)	Link	a (m)	b (m)	α (deg)	θ (deg)
Shoulder Elbow	1	0	0	-90	$\theta_1$
(Joint 3)	2	0.432	0.149	0	$\theta_2$
Wrist rotation (Joint 4)	3	0.02	0	90	θ3
Trunk Z Forearm (Joint 5) Flange (Joint 6)	4	0	0.432	-90	$\theta_4$
y Hange (Joint O)	5	0	0	90	θ <sub>5</sub>
Wrist	6	0	0.056	0	$\theta_6$

# Result: Torque at Joint 1

 $\theta_i = \frac{1}{2} \left[ \frac{2\pi}{T} t - \sin(\frac{2\pi}{T} t) \right]^T T = 10.0 \text{ sec}; \ \theta_i(0) = 0, \text{ and } \theta_i(T) = 180^o$ 



### Comparison

Algorithm	M	A	n = 6		
Hollerbach (1980)	412n - 277	320n - 201	2195M	1719A	
Luh et al. (1980)	150n - 48	131n + 48	852M	834A	
Walker and Orin (1982)	137n - 22	101n - 11	800M	595A	
Proposed	120n - 44	97n - 55	676M	527A	
Khalil et al. (1986)	105n - 92	94n - 86	538M	478A	
Angeles et al. (1989)	105n - 109	90n - 105	521M	435A	
Balafoutis et al. (1988)	93n - 69	81n-65	489M	421A	

#### Forward Dynamics & Simulation

$$\begin{split} \mathbf{\hat{\tau}}_{i} = \mathbf{\hat{v}}_{i} - \mathbf{p}_{i}^{T} \mathbf{\hat{\mu}}_{i,i+1} \quad \mathbf{\hat{\eta}}_{i,i+1} \equiv \mathbf{B}_{i+1,i}^{T} \mathbf{\hat{\eta}}_{i+1} \quad \mathbf{\hat{\eta}}_{i+1} \equiv \mathbf{\hat{\psi}}_{i+1} \hat{\tau}_{i+1} + \mathbf{\hat{\eta}}_{i+1,i+2} \quad \hat{\tau}_{n} \equiv \phi_{n} \\ \mathbf{\hat{\psi}}_{k} \equiv \mathbf{\hat{M}}_{k} \mathbf{p}_{k}, \quad \mathbf{\hat{\psi}}_{ik} \equiv \mathbf{B}_{ki}^{T} \mathbf{\hat{\psi}}_{k}, \\ \mathbf{\hat{\psi}}_{k} \equiv \frac{\mathbf{\hat{\psi}}_{k}}{\hat{m}_{k}}, \quad \text{and} \quad \mathbf{\hat{\psi}}_{ik} \equiv \frac{\mathbf{\hat{\psi}}_{ik}}{\hat{m}_{k}} \quad \mathbf{\hat{T}}_{k} = \mathbf{T}_{i+1,i} \mathbf{\hat{M}}_{i+1,k} = \mathbf{T}_{i+1,i} \mathbf{\hat{M}}_{i+1} = \mathbf{\hat{M}}_{i+1,i} \mathbf{\hat{M}}_{i+1} = \mathbf{\hat{M}}_{i+1,i+2} \quad \mathbf{\hat{T}}_{n} = \mathbf{\hat{M}}_{n} \\ \mathbf{\hat{\mu}}_{i} = \mathbf{\hat{T}}_{i} - \mathbf{\hat{\Psi}}_{i}^{T} \mathbf{\hat{\mu}}_{i,i-1} \quad \mathbf{\hat{\mu}}_{i-1} \equiv \mathbf{p}_{i-1} \mathbf{\hat{\theta}}_{i-1} + \mathbf{\hat{\mu}}_{i-1,i-2} \quad \mathbf{\hat{\mu}}_{i,i-1} \equiv \mathbf{B}_{i,i-1} \mathbf{\hat{\mu}}_{i-1} \quad \mathbf{\hat{\mu}}_{1,0} = \mathbf{0} \\ \mathbf{\hat{\Psi}}_{k} \equiv \mathbf{\hat{M}}_{k} \mathbf{p}_{k}, \quad \mathbf{\hat{\Psi}}_{ik} \equiv \mathbf{B}_{ki}^{T} \mathbf{\hat{\Psi}}_{k}, \\ \mathbf{\hat{M}}_{k} = \mathbf{M}_{i} + \mathbf{B}_{i+1,i}^{T} \mathbf{\hat{M}}_{i+1,k} \mathbf{B}_{i+1,i} \end{split}$$

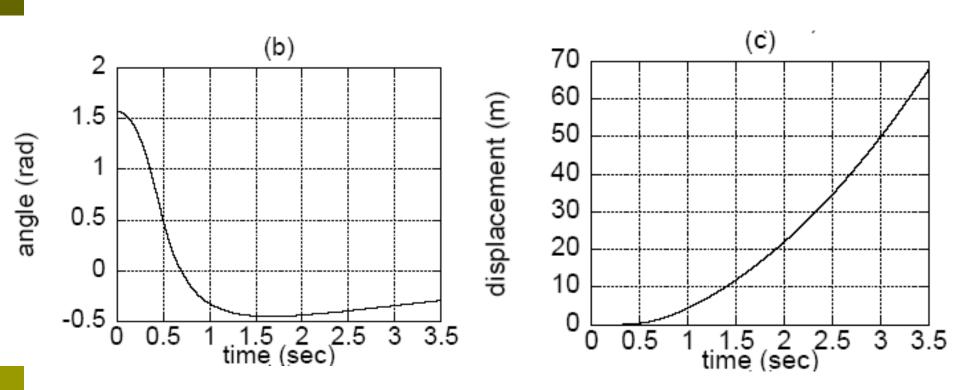


Algorithm	M	A
Proposed	191n - 284	187n - 325
Featherstone (1983)	199n - 198	174n - 173
Valasek <sup>†</sup>	226n - 343	206n - 345
Brandl et al. †	250n - 222	220n - 198
Walker and Orin (1982)	$\frac{1}{6}n^3 + 11\frac{1}{2}n^2$	$\frac{1}{6}n^3 + 7n^2$
	$+38\frac{1}{3}n-47$	$+38\frac{5}{6}n-46$

n = 6	n = 10
<u>862M 797A</u>	$1626M \ 1545A$
$996M \ 871A$	$1792M \ 1567A$
$1013M \ 891A$	$1917M \ 1715A$
$1278M \ 1122A$	$2278M \ 2002A$
$633M \ 480A$	$1653M \ 1209A$

Results: Stanford Robot						Not	End-effec	ctor $\downarrow$ $\theta_{6}$ $Z_{e} Z_{6}$ , $X_{e}$ $Y_{e}$ , $Z_{5}$ $Z_{4}$ , $\theta_{4}$ $\theta_{4}$ $b_{3}$	θ <sub>5</sub> .6m		
DI	DH and Inertia Parameters							$\theta_1$ $X_2$ $X_1$ Bas	θ <sub>2</sub> .1m	Z 2 X3.	X4 Im
i	$a_i$	$b_i$	$\alpha_i$	$\theta_i$	$m_i$	$r_x$	$r_y$	$r_z$	$I_{xx}$	$I_{yy}$	$I_{zz}$
	(m)	(m)	(deg)	(deg)	(kg)		(m)	(kg-m <sup>2</sup> )			
1	0	.1	-90	$\theta_1[0]$	9	0	1	0	.01	.02	.01
<b>2</b>	0	.1	-90	$\theta_2[90]$	6	0	0	0	.05	.06	.01
3	0	$b_{3}[0]$	0	0	4	0	0	0	.4	.4	.01
4	0	.6	90	$\theta_4[0]$	1	0	1	0	.001	.001	.0005
5	0	0	-90	$\theta_5[0]$	.6	0	0	0	.0005	.0005	.0002
6	0	0	0	$\theta_6[0]$	.5	0	0	0	.003	.001	.002

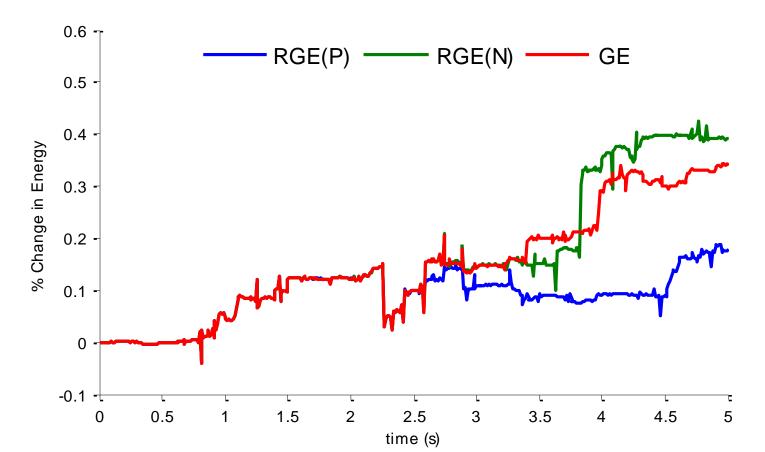
#### Free-Fall: Joints 2 and 3



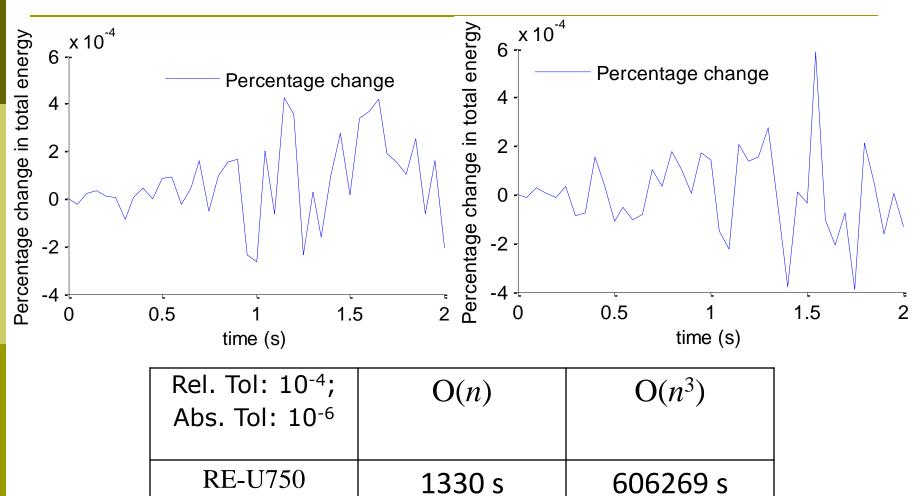
Robot Software: <u>RoboAnalyzer</u> www.roboanalyzer.com

### Why **UDU**<sup>T</sup>?

#### Accurate (based on Reverse Gaussian Elimination) [Plot for 3-link system]



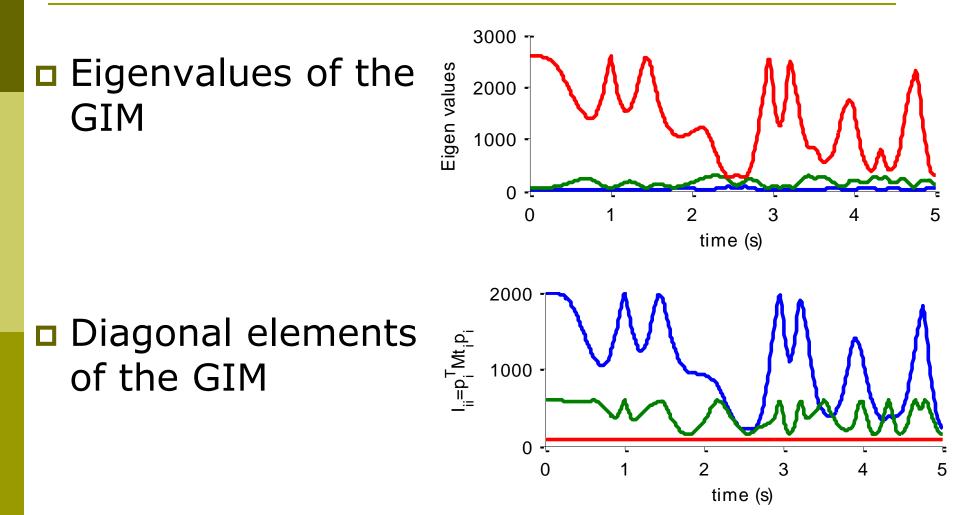
#### Efficient

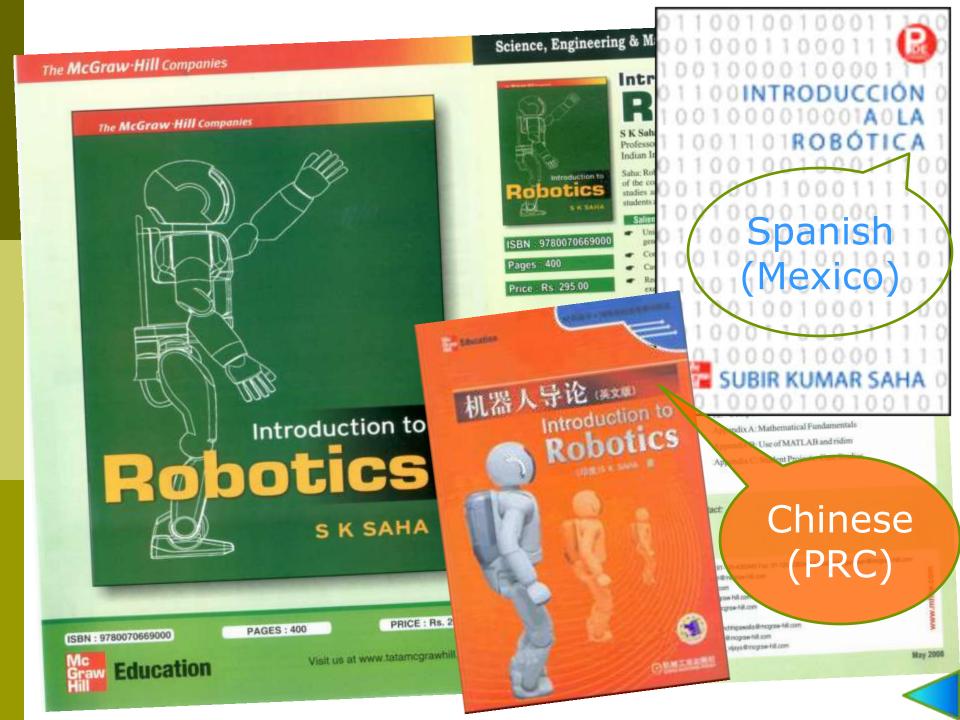


(22min)

(7 days)

#### Information on Stability





## Closed-loop Systems

8

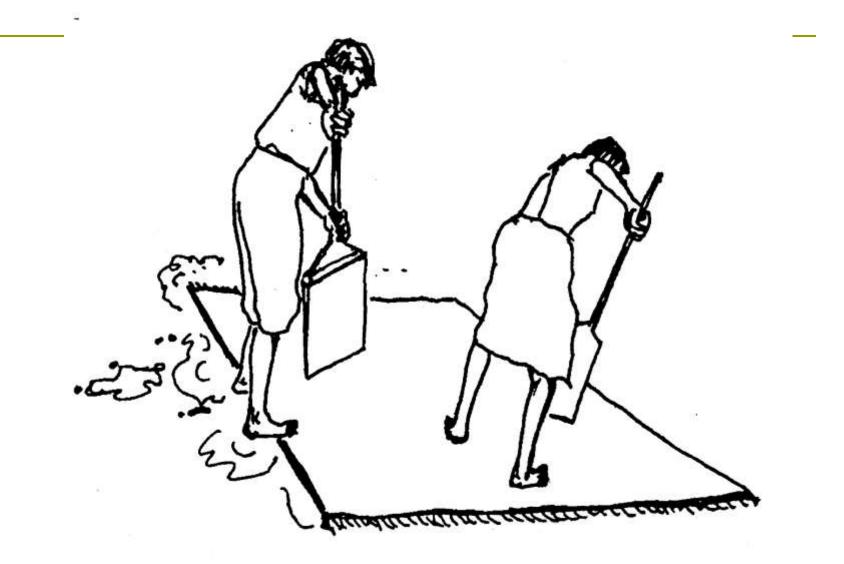
5

6

3

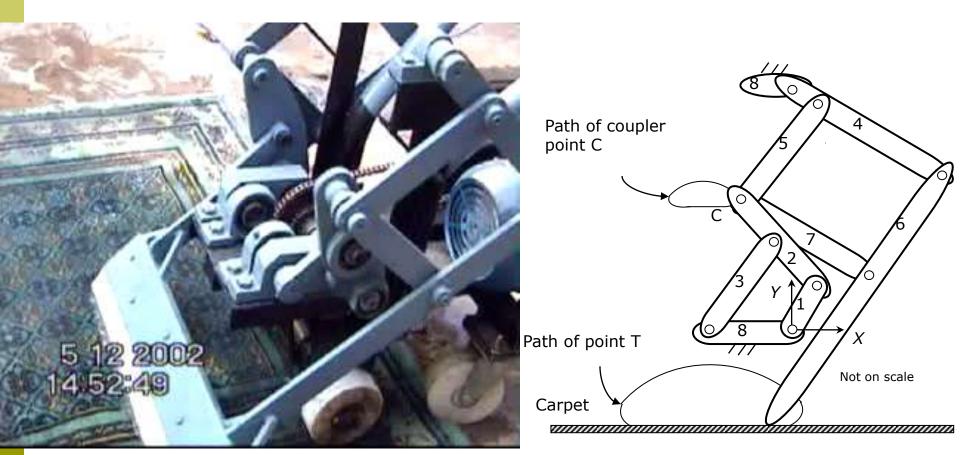
- A Mechanism used for walking legs
- Consisting of two closed-loop linkages
- It has several loops
- Applied also in Carpet Scrapper

#### Carpet Cleaning: Traditional



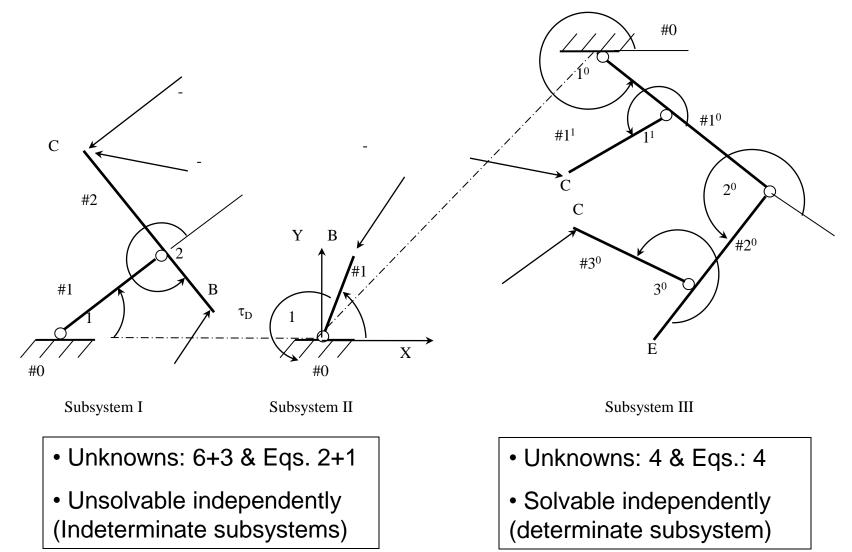
#### Carpet Scrapping Machine

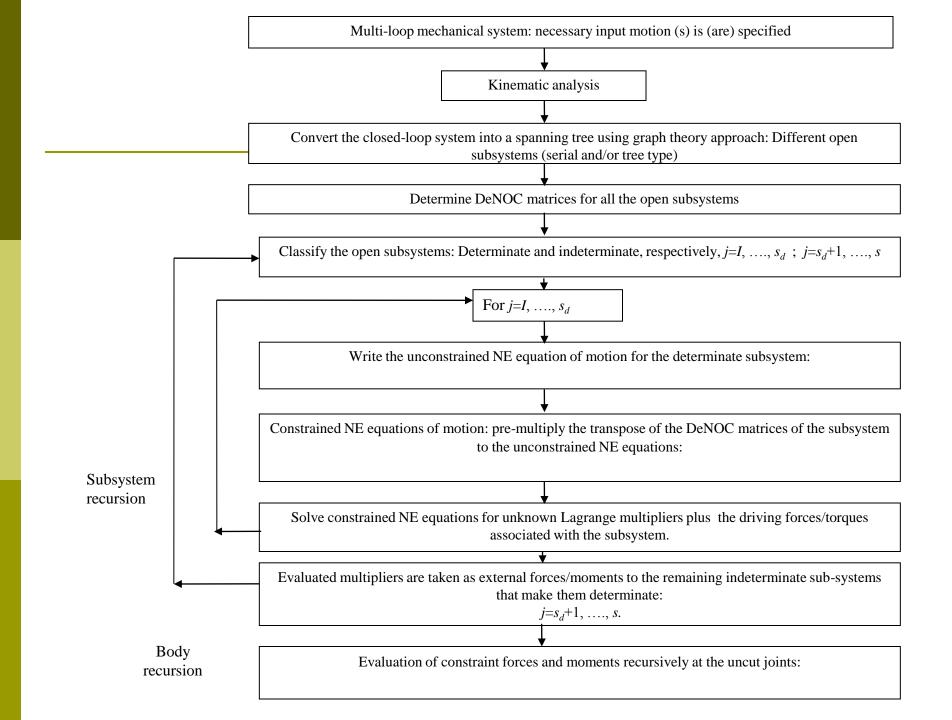
# Purpose: To reduce human effortStraight line generating machine

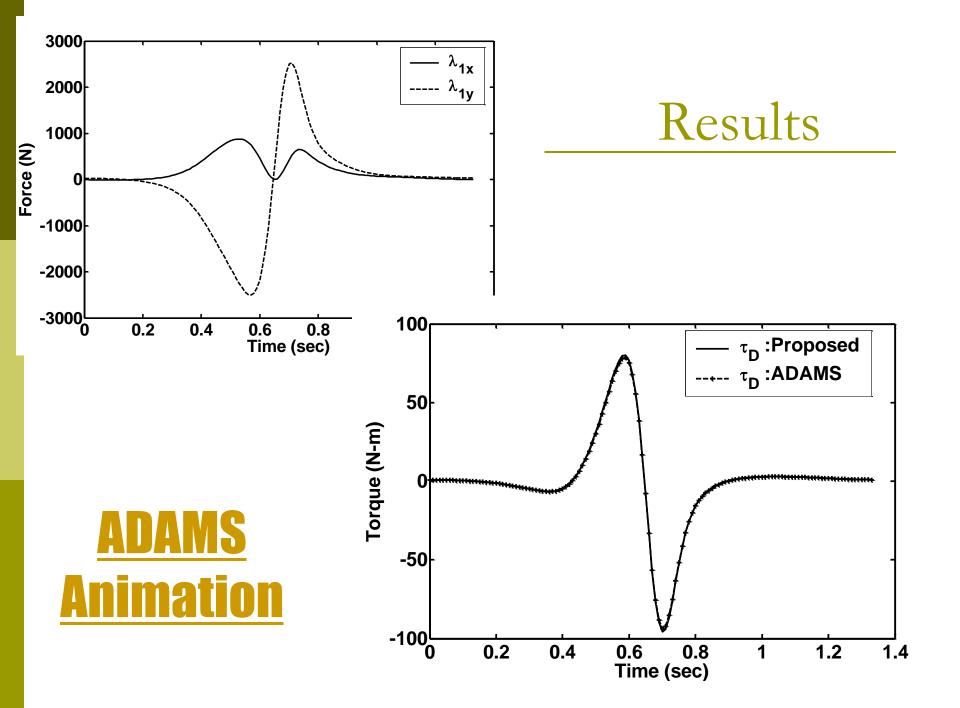


## Tree-types: Double Recursion

#### ASME J. of Mech. Des., Dec. 2007



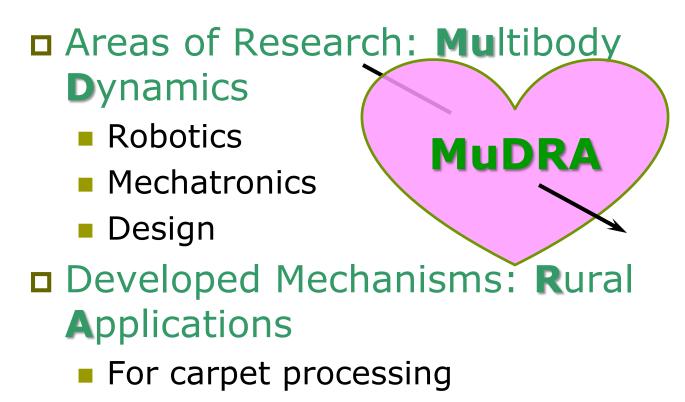




## Comparison

Methods	Theoretical order of computations	CPU time in sec
Traditional (matrix size: 21×21)	O(21 <sup>3</sup> /3)=O(3087)	0.219
System approach (matrix size: 7×7)	O(7 <sup>3</sup> /3)+O(7×2)=O(128.3)	0.156 (30.59)
Subsystem approach (matrix sizes: 4×4, 3×3, 1×1)	$O(4^{3}/3+3^{3}/3+1)+O(7\times2)=O(45.3)$	0.156 (30.59)

Multibody Dynamics for Rural Applications (MuDRA)



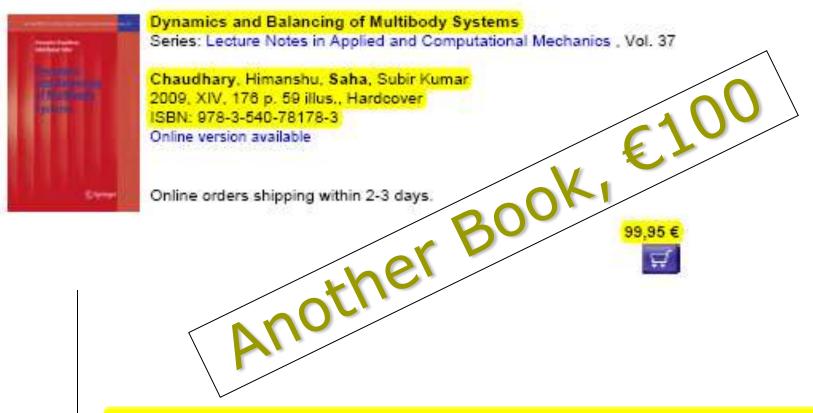
For villages ADPM

## MuDRA Concept

Floated as B. Tech/M. Tech Projects Not interested Apparent Reasons What is the research content ? Not fashionable Other Reasons Limited literature Difficult



#### springer.com



This book has evolved from the passionate desire of the authors in using the modern concepts of multibody dynamics for the design improvement of the machineries used in the rural sectors of India and The World. In this connection, the first author took up his doctoral research in 2003 whose findings have resulted in this book. It is expected that such developments

# **Every drop counts**

PETER FRYKMAN, A STUDENT OF STANFORD UNIVERSITY, HAS INVENTED A MANUFACTURING TECHNOLOGY TO HELP MAKE AFFORDABLE DRIP IRRIGATION FOR SMALL-PLOT FARMERS IN DEVELOPING COUNTRIES. MALINI SEN REPORTS

third of the world's population suffers from water scarci-Lty. Without access to affordable water efficient irrigation, smallplot farmers are unable to grow crops during much of the year.

As part of a Stanford University course in Entrepreneurial Design for Extreme Affordability, Peter Frykman travelled to Ethiopia, where he witnessed first-hand the hardships caused by the worst drought Ethiopia had experienced in 20 years. The farmers he met had no means to grow grons with their

scarce water resources. Locally available drip irrigation products were too expensive and seldom worked properly.

Recognising the need for less costly and more effective ways for smallplot farmers to use their meagre water supplies efficiently, Frykman returned to Stanford and invented a manufacturing technology, and launched his company, Driptech, which makes affordable drip irrigation for small-plot farmers in developing countries.

Describing the technology behind Driptech, Frykman says there are two important parts to the technology. "The first part is the attention we paid to small-scale farmers and how they work that allowed us to design the system specifically to meet their needs. We learned the importance of making our systems simple in order to reduce installation and mainte-





Stanford graduate Peter Frykman explains his invention to farmers in India

nance costs. We learned that highly uniform water application is essential, even with low pressure, so we

designed Driptech tubing with the very precise, CAS clean, consistent holes that

small plots come in many configurations, and made our system modular by designing parts that continue to function well even if it scales up or down," he elaborates.

The second part is that Driptech is able to manufacture this product

anywhere in the world using existing plastic bag machinery. "We developed

make this possible. We learned that small, inexpensive machines that reliably produce tubing to the high level of quality that we require. This 'distributed manufacturing' model allows us to customise products

to meet local needs, minimise transportation costs, and create jobs in nearby communities," adds Frykman.

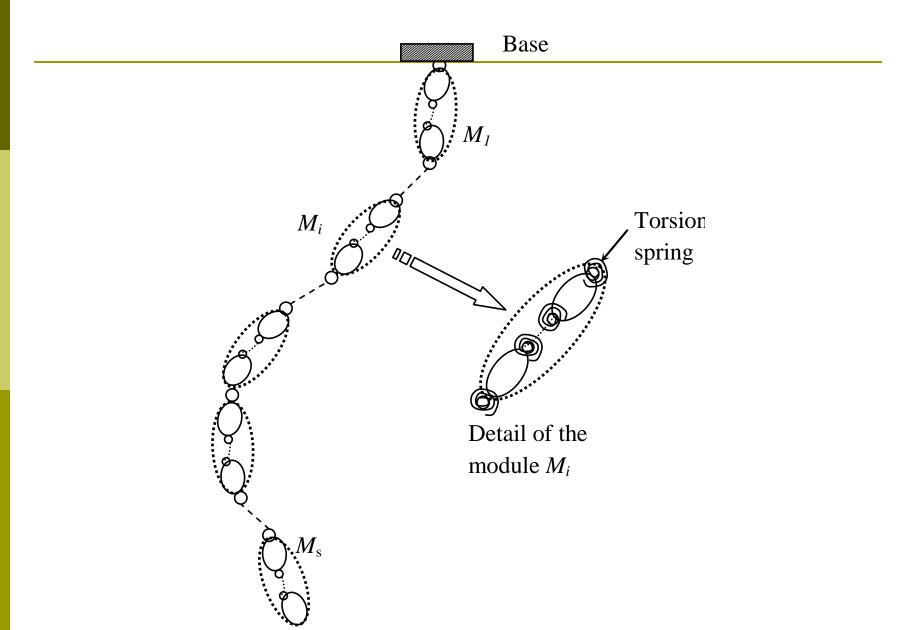
Compared to flood irrigation, Driptech irrigation increases crop vields by 20% to 90%, improves product quality, saves water by 30% to 70%, and reduces the required labour, energy costs for pumping, and fertiliser. Driptech systems operate on very low water pressures as well.

As a graduate student in mechanical engineering at Stanford, Frykman attended the Summer Institute for Entrepreneurship, and he found the programme "exhilarating" and "highly beneficial."

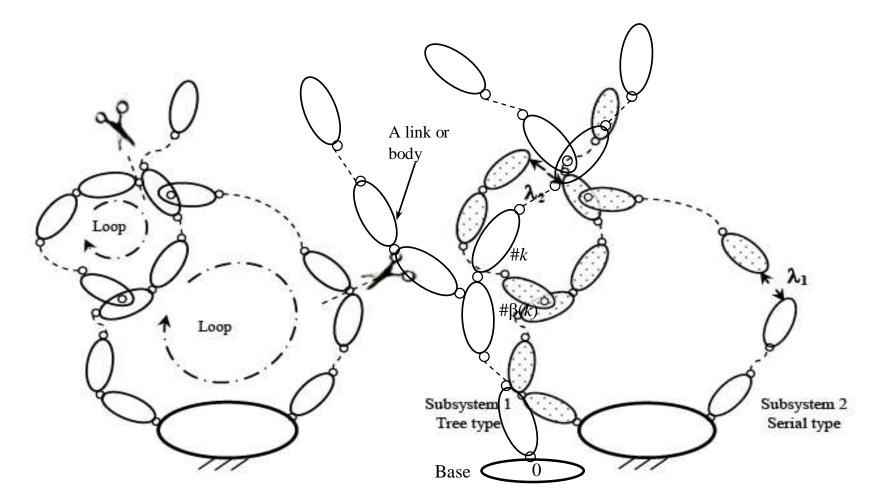
"I applied for the programme because I needed help developing my underlying passion for entrepreneurship. I relied on lessons that I had learned during the programme to launch my company Driptech. Even without a formal business background I had the confidence to launch my own venture

Currently Frykman and his team are working with farmers in Maharashtra and Karnataka and last year. Driptech launched commercially in India by partnering with one of the largest manufacturing and retail conglomerates in the country. Driptech's systems sell directly to farmers through retail and local distribution channels.

#### <u>Ropes</u> (Courtesy: Dr. Suril V. Shah)



## Multibody Systems: <u>ReDySim</u>



## Biological Systems: Study of Proteins

Proteins

(Nanoparticles: 1– 100 nm) are part of every cell, tissue, and organ in our bodies.

Body proteins are constantly being broken down and replaced. The protein in the foods we eat is digested into amino acids that are later used to replace these proteins in our bodies.

#### Protein

- Proteins are made up of amino acids.
- Think of amino acids as the building blocks.
- There are 20 different amino acids that join together to make all types of protein.

Some of these amino acids can't be made by our bodies, so these are known as essential amin o acids.

It's essential that our diet provide these. "Analysis and Design of Protein Based Nanodevices: Challenges and Opportunities in Mechanical Design" by <u>Gregory S. Chirikjian</u> [ASME J. Mech. Design, July 2005, Vol. 127]

- Each of these amino acid building blocks (monomers) is referred to as a residue.
- Tens to hundreds of these residues amino acid monomers connect together in a serial manner to create a long chain, known as a polypeptide chain.

#### Protein vs. Serial Robot

- From a kinematics point of view, these polypeptide molecules can be considered to be a chain of miniature rigid bodies connected by revolute hinge joints.
- A protein in its denatured state is a serial linkage with N +1 solid links connected by N revolute joint values for N could be as great as several hundred.

## Protein Folding

- Protein folding is the process by which a protein structure assumes its functional shape or conformation.
- The folding occurs under the effect of nuclear forces among protein atoms as well as between protein atoms and the solvent's atoms.
- Can be studied through dynamic simulation

#### Result

Being able to accurately predict the threedimensional structure of a protein based on the known sequences of amino acids in its chain is key to fully understanding a protein's biological functions and thus to manipulating or controlling these functions as a part of disease treatments.

### Activities with Chair Professor Fund

Presenting new research at ACMD 2010 in Japan

- Research collaboration with IIT Madras on Rope Modeling
- Supporting several Students/Faculty to visit IIT Delhi and vice-verse

Will be attending ICMD 2012 in Germany

## Acknowledgements

#### MR. NAREN GUPTA Dr. Sandipan

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- Mr. Rajeev Lochan
- Mr. Amit Jain
- Ms. Jyoti Bahuguna
- Mr. D. Jaitly, and others

- Dr. Sandipan Bandyopadhyah, IITM
- Dr. Madhav Krishnan, IIITH
- Mr. Vijay, IIITH

#### Conclusions

- DeNOC for serial-chain systems
- RoboAnalyzer
- Closed-loop systems and optimization
- MuDRA
- Rope
- New application to biological systems

## THANK YOU for Your Attention

http://web.iitd.ac.in/~saha saha@mech.iitd.ac.in