

# Recursive Robot Dynamics in RoboAnalyzer

C. G. Rajeevlochana, A. Jain, S. V. Shah, S. K. Saha

## Abstract

Robotics has emerged as a major field of research and application over the years, and has also found a place in the curriculum of universities. Robotics as a course is challenging both for the teachers to teach and the students to learn as it involves 3D transformations, algebraic and differential equations, etc., which are difficult to understand. Several robotics learning software have been developed have helped to ease the learning of robotics as a subject. A similar attempt was made in developing RoboAnalyzer, a 3D model based robotics learning software that modelled a serial robot based on its DH-parameters. It could perform forward kinematics and show animation and graph plot as outputs. In this paper, further development of RoboAnalyzer is reported in the form of addition of inverse and forward dynamics analyses of a generic serial manipulator. The important contributions of this paper lie in the development of algorithms using an object oriented modelling approach and the Decoupled Natural Orthogonal Complement (DeNOC)-based recursive formulation. A KUKA KR5 robot was modelled in the proposed software, and the results were verified with those obtained using the Dynamic Simulation module of Autodesk Inventor. RoboAnalyzer can be downloaded for free from <http://www.roboanalyzer.com> and can be used almost instantly.

**Keywords:** DeNOC, DH Parameters, Recursive Robot Dynamics, Robot Analysis, Robotics Learning Software.

## 1 Introduction

Robotics has evolved from a research interest to a widely used field today. It is related to design, development, control and application of robots. It finds its application in several industries such as automobile, electronics, medical healthcare and space. As a result, it has gained enormous importance in the recent past and finds its place in curriculum of universities.

Generally, the mathematics involved in the study of robotics, e.g., kinematics, dynamics, etc. is difficult to understand by students and difficult to explain by a teacher. It involves 3D transformations, solution of algebraic equations in kinematics and solution of differential equations in dynamics that are complex in nature. In order to make learning and teaching of robotics easier, several software/toolkits have been developed over a period of time. They are in the form of a desktop application using OpenGL [1-2], toolkit for MATLAB [3-4], LabVIEW [5] and as an internet service using Virtual Reality Markup Language (VRML) [6-7]. However, most of them cater only to certain examples of serial manipulators and do not give flexibility to the end user to change the manipulator's architecture according to his/her needs [8]. Commercial multibody system software like ADAMS, DADS, RecurDYN, Autodesk Inventor, etc. on the other hand provide a lot of flexibility, however,

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require users to develop the CAD model of the robot first before they can analyze it. To ease out the above mentioned difficulty for students and teachers, RoboAnalyzer is being developed as an independent desktop application and it encourages learning the physics of a robot's motion first before learning the underlying mathematics. In the earlier version of RoboAnalyzer [8], it could model a serial manipulator based on its representation using the well-known Denavit and Hartenberg (DH) [9] parameters. One could perform forward kinematics and see animation as shown in Fig. (1) and view the graph plots as output.

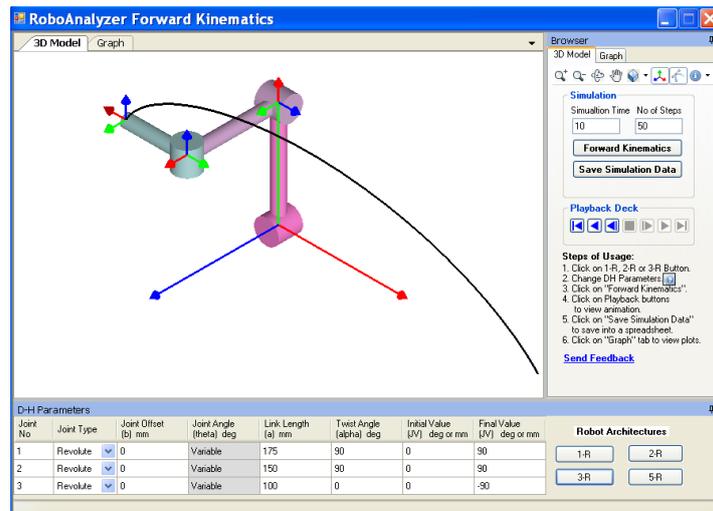


Figure 1: RoboAnalyzer animation of a 3 degree-of-freedom serial manipulator

In this paper, dynamics modules of RoboAnalyzer are introduced. For that, the paper is organized as follows: Section 2 explains RoboAnalyzer and its dynamics algorithms; Section 3 illustrates the use of RoboAnalyzer in analyzing the dynamics of an industrial robot, KUKA KR5 and its validation with the results obtained using the Dynamic Simulation module of Autodesk Inventor. Finally, conclusions are given in Section 4.

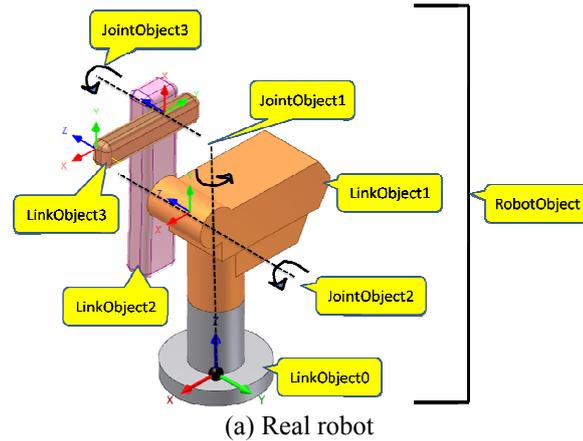
## 2 RoboAnalyzer

Concept of Object Oriented Modeling (OOM) has become the de-facto standard of software development [10]. It is used in most engineering software applications and is considered to be effective for a multibody dynamics software [11-14] as the real life objects such as robot, links and joints can be mirrored by respective software objects as shown in Fig. (2), thus helping in robust development and maintenance of the software. For the development of RoboAnalyzer, Visual C# programming language was chosen for various merits mentioned in [15].

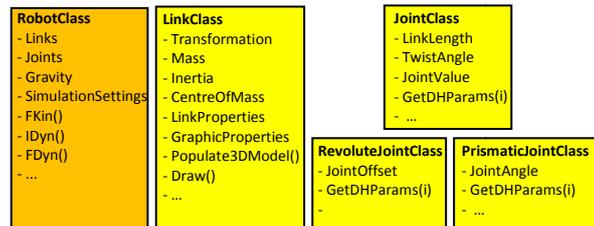
### 2.1 Robot Dynamics

The Decoupled Natural Orthogonal Compliment (DeNOC) [16-18]-based approach allows one to obtain the recursive order ( $n - 'n'$  being the number of links in a

robotic system – dynamics algorithms. Each scalar element of the matrices and vectors associated with the equations of motion can be written analytically, which allows one to provide the physical interpretations like the composite and articulated body inertia, etc., and helps a programmer to debug the computer algorithms. Since the approach is built upon the basic mechanics and linear algebra theories, it can be



(a) Real robot



(b) Software objects

Figure 2: Real object components vs. software objects

easily comprehended. In view of the above advantages, the DeNOC-based approach is used in RoboAnalyzer.

Dynamics comprises of two basic problems, namely, inverse dynamics and direct or forward dynamics. Given the time history of motion characteristics of the system and the forces and moments acting on it, finding the actuator forces/torques required to maintain the given motion characteristics is known as inverse dynamics (IDyn). On the other hand, given the initial motion state of the system and the time history of the driving forces/torques at the actuated joints, determining the resulting motion history of the system at later time instant is known as forward dynamics (FDyn).

The IDyn module of a serial manipulator in RoboAnalyzer takes the inputs in the form of robot’s DH parameters, mass, inertia and centre of mass of all the links, direction of the gravity and simulation settings. For the given time history of the joint values, IDyn calculates the joint torques/ forces of a given serial manipulator recursively [19], as shown in Fig. (3) in which all variables are defined in [20].

In addition to the inputs of IDyn module, FDyn module takes initial values of joint position and velocity of each joint. For the given time history of each joint generalized force, FDyn finds the joint accelerations recursively, as explained in Fig. (4). The accelerations are then passed to Runge-Kutta (RK4) [21] numerical

integrator developed for the purpose of using in RoboAnalyzer in Visual C# [19] to determine the joint velocities and positions. On completion of FDyn analysis, the animation of the robot motion and graph plots of the joint values can be viewed as outputs.

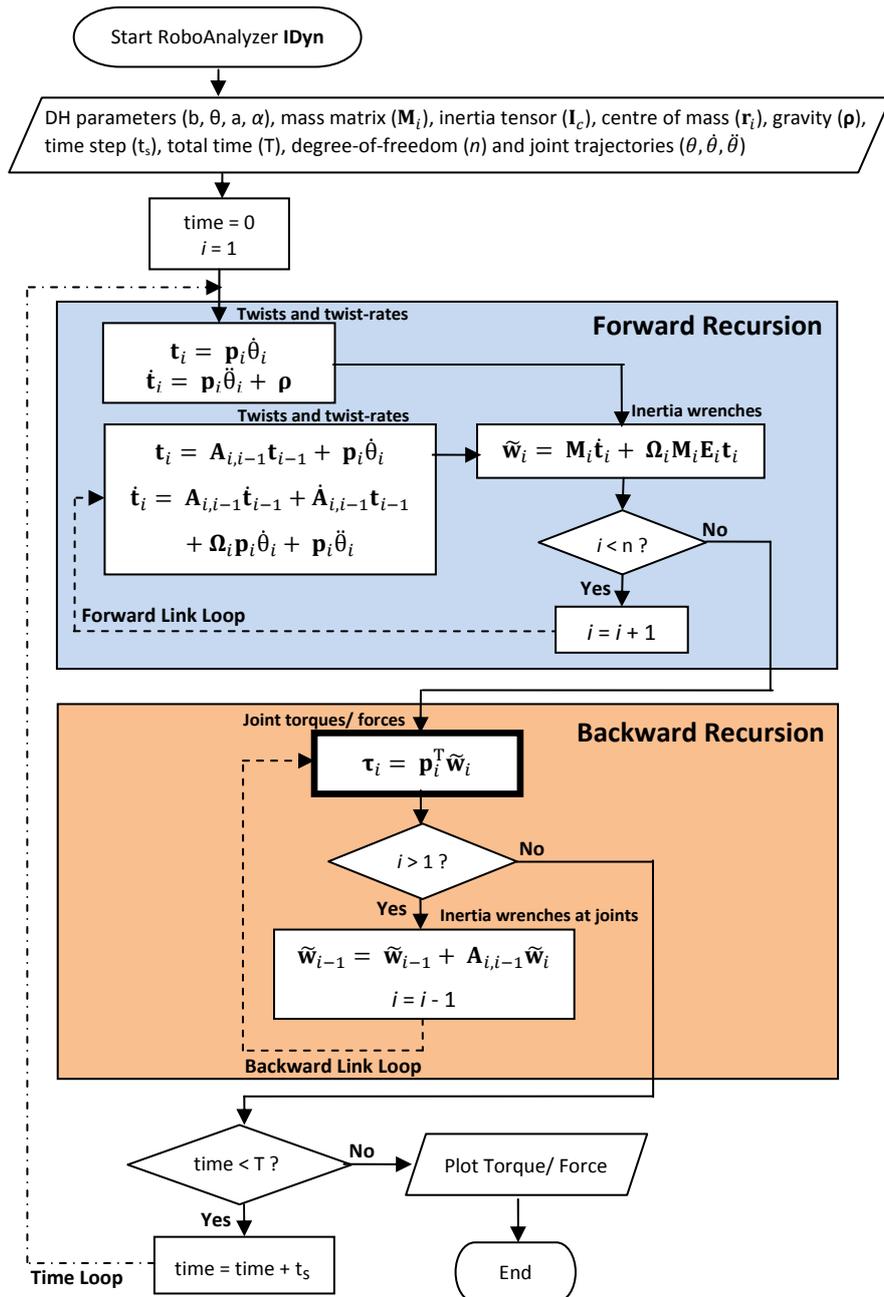


Figure 3: Flowchart of IDyn algorithm

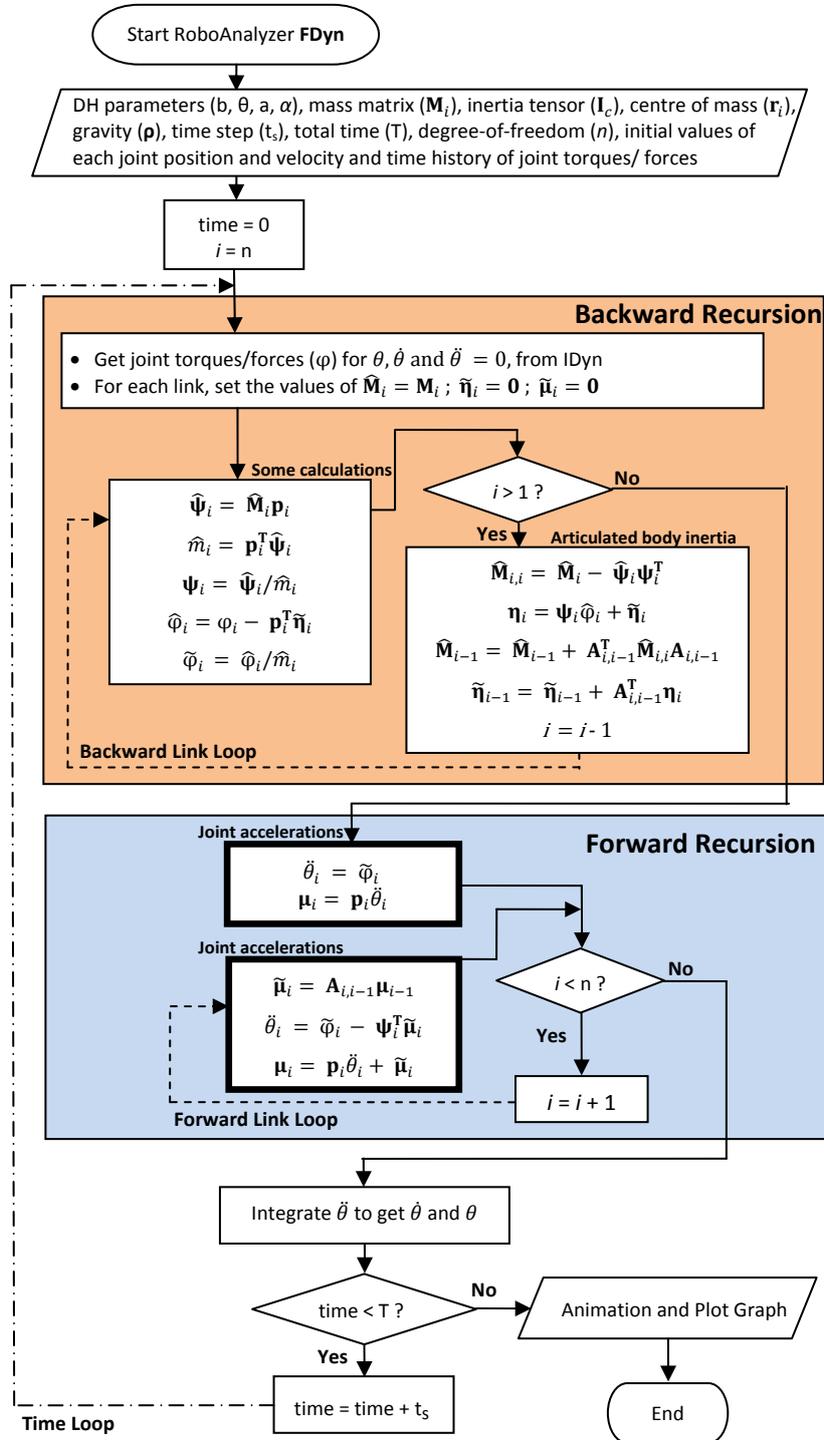


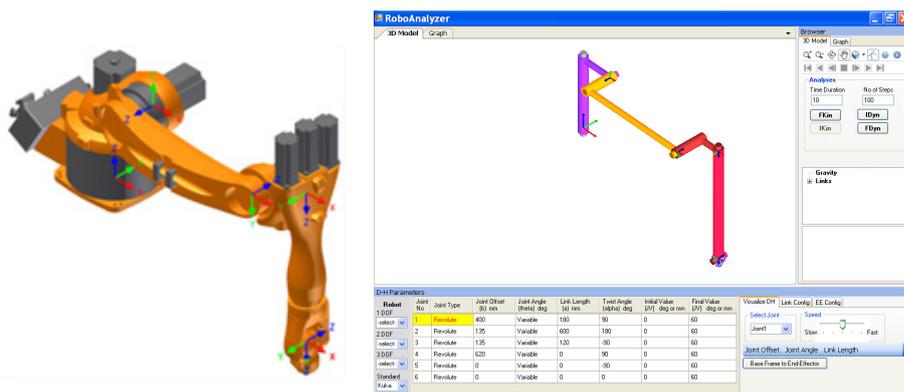
Figure 4: Flowchart of FDyn algorithm

### 3 An Illustration: KUKA KR5

The IDyn and FDyn modules were developed for RoboAnalyzer to carry out inverse and forward dynamics of various planar and spatial serial manipulators with revolute and prismatic joints. In this paper, the analyses of a 6-axis industrial robot, namely, KUKA KR5 [22], is reported.

#### 3.1 The DH parameters of KUKA KR5

CAD model of KUKA KR5 robot was imported [22] and assembled in Autodesk Inventor software, as shown in Fig. (5a). The co-ordinate frames were attached to the robot links and the DH parameters were determined as given in Table 1. These parameters were given as inputs to RoboAnalyzer and the 3D model of KUKA KR5 robot was generated, as shown in Fig. (5b). The mass and inertia properties of all the robot links were retrieved from the Autodesk Inventor software and then put as inputs to RoboAnalyzer for the dynamic analyses.



(a) Assembly in Autodesk Inventor      (b) 3D model in RoboAnalyzer  
Figure 5: KUKA KR5 robot

Table 1: DH parameters of KUKA KR5

| Joint | <b>b (mm)</b> | <b><math>\theta</math> (degree)</b> | <b>a (mm)</b> | <b><math>\alpha</math> (degree)</b> |
|-------|---------------|-------------------------------------|---------------|-------------------------------------|
| 1     | 400           | 0 (variable)                        | 180           | 90                                  |
| 2     | 135           | 0 (variable)                        | 600           | 180                                 |
| 3     | 135           | 0 (variable)                        | 120           | -90                                 |
| 4     | 620           | 0 (variable)                        | 0             | 90                                  |
| 5     | 0             | 0 (variable)                        | 0             | -90                                 |
| 6     | 0             | 0 (variable)                        | 0             | 0                                   |

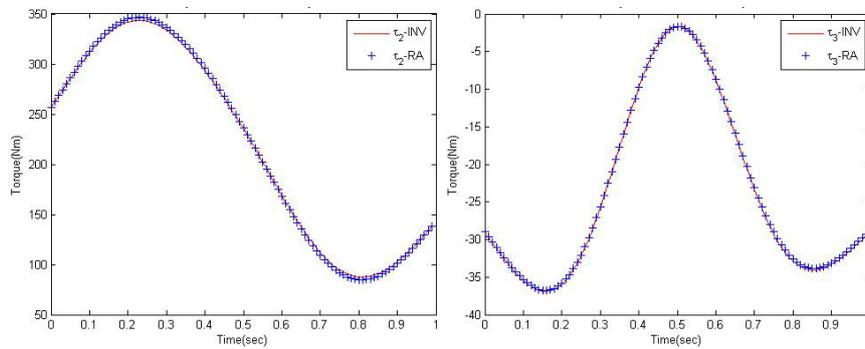
#### 3.2 Inverse dynamics of KUKA KR5

The inverse dynamics was performed using the IDyn module of RoboAnalyzer for all the joint angles of KUKA KR5 varying from 0° to 60° in a cycloidal fashion [23] as given by Eq. (1):

$$\theta_i(t) = \theta_i(0) + \frac{\theta_i(T) - \theta_i(0)}{T} \left[ t - \frac{T}{2\pi} \sin\left(\frac{2\pi}{T}t\right) \right] \quad (1)$$

where  $\theta_i$  is the joint variable,  $t$  is the current time and  $T$  is the total time.

The joint torques for joints 2 and 3 only are shown in Figs. (6a) and (6b) respectively. Other joint torques are not shown due to space limitations. However, all the joint torques were compared with those obtained in the Autodesk Inventor as indicated in Figs. (6a-b) as “INV”, whereas the results from RoboAnalyzer are shown as “RA”.



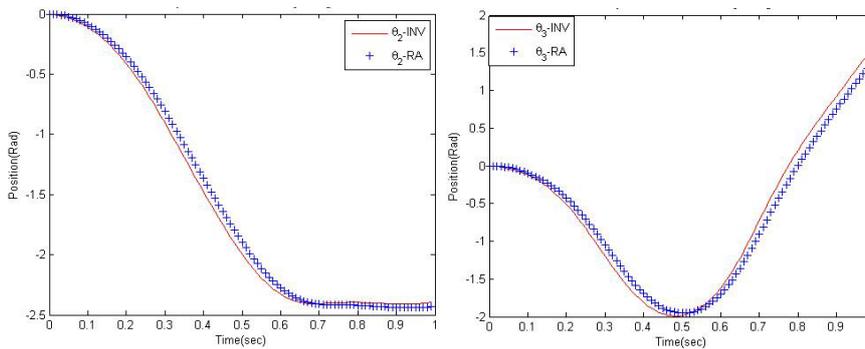
(a) Joint 2

(b) Joint 3

Figure 6: IDyn results of KUKA KR5

### 3.3 Forward dynamics of KUKA KR5

The FDyn module of RoboAnalyzer was used to study the free-fall simulation of KUKA KR5 under the action of gravity. The variation of the joint angles for joints 2 and 3 are shown in Fig. (7), whereas the positions along X-and-Z directions are shown in Fig. (8). The results are compared with those obtained in Dynamic Simulation module in Autodesk Inventor for the same initial conditions.



(a) Joint 2

(b) Joint 3

Figure 7: FDyn results of KUKA

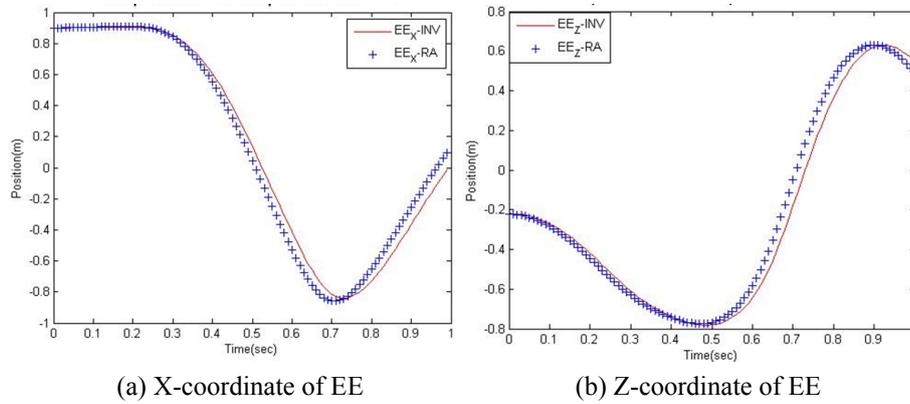


Figure 8: The end-effector (EE) positions

## 4 Conclusions

An attempt to make robotics learning easier was made in the form of RoboAnalyzer. Inclusion of the inverse and forward dynamics modules for generic serial manipulators in RoboAnalyzer is reported in this paper which will make students understand the dynamic behaviour without going through the details of the dynamic formulation. As a result, even an undergraduate student will appreciate the physics and will be capable of designing a better robot for any practical purpose. To widen the use of the software, it has been made available free through <http://www.roboanalyzer.com>.

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## References

- [1] X. Yang, Y. Zhao, W. Wu and H. Wang, “Virtual reality based robotics learning system,” IEEE International Conference on Automation and Logistics, pp.859-864, 2008.
- [2] M. A. González-Palacios, E. A. González-Barbosa and L. A. Aguilera-Cortés, “An interactive software package for the simulation of serial manipulators,” ECTC Proceedings, ASME Early Career Technical Conference, 2009.
- [3] P. I. Corke, “A robotics toolbox for MATLAB,” Robotics & Automation Magazine, IEEE, vol.3, no.1, pp.24-32, 1996.
- [4] M. Toz and S. Kucuk, “Dynamics Simulation Toolbox for Industrial Robot Manipulators,” Computer Applications in Engineering Education, 18: 319–330. doi: 10.1002/cae.20262, 2010.

- [5] T. J. M. Sanguino and J. M. A. Márquez, "Simulation Tool for Teaching and Learning 3D Kinematics Workspaces of Serial Robotic Arms with up to 5-DOF," *Computer Applications in Engineering Education*, doi: 10.1002/cae.20433, 2010.
- [6] M. F. Robinette, and R. Manseur, "ROBOT-DRAW, an Internet-Based Virtualization Tool for Robotics Education," *IEEE Transactions on Education*, Vol. 44, No. 1, 2001.
- [7] R. Manseur, "Modeling and Visualization of Robotic Arms," *Proceedings of IASTED International Conference on Graphics and Visualization in Engineering*, 2007.
- [8] C. G. Rajeevlochana and S. K. Saha, "RoboAnalyzer: 3D Model Based Robotic Learning Software," *Proceedings of International Conference on Multi Body Dynamics*, pp. 3-13, 2011.
- [9] J. Denavit, and R. S. Hartenberg, "A kinematic notation for lower-pair mechanisms based on matrices," *ASME Journal of Applied Mechanisms*, pp. 215–221, 1955.
- [10] G. Engels and L. Groenewegen, "Object-oriented modeling: a roadmap". *The Future of Software Engineering, ICSE*, 2000.
- [11] A. S. Koh and J. P. Park, "Object oriented dynamics simulator," *Computational Mechanics*, vol. 14, pp. 277-287, 1994.
- [12] A. Kecskemethy and M. Hiller, "An object-oriented approach for an effective formulation of multibody dynamics," *Computer Methods in Applied Mechanics and Engineering*. Vol. 115, pp. 287–314, 1994.
- [13] D. Kunz, "An object-oriented approach to multibody systems analysis," *Computers & Structures*, vol. 69, pp. 209-217, 1998.
- [14] H. S. Han and J. S. Seo "Design of a multi-body dynamics analysis program using the object-oriented concept," *Advances in Engineering Software*, vol. 35, pp. 95-103, 2004.
- [15] K. Hoffman, *Microsoft Visual C#*, SAMS Publishers, Indiana USA, 2005.
- [16] S. K. Saha, "A decomposition of the manipulator inertia matrix," *IEEE Transactions on Robotics and Automation* Vol. 13, No. 2, pp. 301-304, 1997.
- [17] S. K. Saha, "Dynamic modelling of serial multi-body systems using the decoupled natural orthogonal complement matrices," *ASME Journal of Applied Mechanics*, Vol. 66, pp. 986-996, 1999.
- [18] S. K. Saha, "Simulation of industrial manipulators based on the  $UDU^T$  decomposition of inertia matrix," *International Journal of Multibody System Dynamics*, Vol. 9, No. 1, pp. 63-85, 2003.
- [19] A. Jain, *Development of a CAD-based Tool for the Study of Robotics*. M.Tech thesis, IIT Delhi, 2011.
- [20] S. V. Shah, S. K. Saha and J. K. Dutt, "Modular recursive dynamics algorithm for multibody systems using the DeNOC matrices," *International Conference on Multibody Dynamics (ECCOMAS Thematic Conf.)*, pp. 1-20, 2009.
- [21] D. F. Griffiths and D. J. Higham, *Numerical Methods for Ordinary Differential Equations - Initial Value Problems*, Springer Undergraduate Mathematics series, London, 2010.
- [22] KUKA Robotics website, <http://www.kuka.com>
- [23] S. K. Saha, *Introduction to Robotics*, Tata McGraw Hill, New Delhi, 2008.